1. (a) The angular wave number is $k = \frac{2\pi}{\lambda} = \frac{2\pi}{1.80\,\text{m}} = 3.49\,\text{m}^{-1}$.

(b) The speed of the wave is $v = \lambda f = \frac{\lambda\omega}{2\pi} = \frac{(1.80\,\text{m})(110\,\text{rad/s})}{2\pi} = 31.5\,\text{m/s}$.

2. The distance $d$ between the beetle and the scorpion is related to the transverse speed $v_t$ and longitudinal speed $v_\ell$ as

$$d = v_t t_t = v_\ell t_\ell$$

where $t_t$ and $t_\ell$ are the arrival times of the wave in the transverse and longitudinal directions, respectively. With $v_t = 50$ m/s and $v_\ell = 150$ m/s, we have

$$\frac{t_t}{t_\ell} = \frac{v_\ell}{v_t} = \frac{150 \text{ m/s}}{50 \text{ m/s}} = 3.0 .$$

Thus, if

$$\Delta t = t_t - t_\ell = 3.0 t_\ell - t_\ell = 2.0 t_\ell = 4.0\times10^{-3} \text{ s} \quad \Rightarrow \quad t_\ell = 2.0\times10^{-3} \text{ s} ,$$

then $d = v_\ell t_\ell = (150 \text{ m/s})(2.0\times10^{-3} \text{ s}) = 0.30 \text{ m} = 30$ cm.

3. (a) The motion from maximum displacement to zero is one-fourth of a cycle so 0.170 s is one-fourth of a period. The period is $T$ = 4(0.170 s) = 0.680 s.

(b) The frequency is the reciprocal of the period:

$$f = \frac{1}{T} = \frac{1}{0.680\,\text{s}} = 1.47\,\text{Hz}.$$

(c) A sinusoidal wave travels one wavelength in one period:

$$v = \frac{\lambda}{T} = \frac{1.40\,\text{m}}{0.680\,\text{s}} = 2.06\,\text{m/s}.$$

4. (a) The speed of the wave is the distance divided by the required time. Thus,

$$v = \frac{853\,\text{seats}}{39\,\text{s}} = 21.87\ \text{seats/s} \approx 22\ \text{seats/s}\,.$$

(b) The width $w$ is equal to the distance the wave has moved during the average time required by a spectator to stand and then sit. Thus,

$$w = vt = (21.87\ \text{seats/s})(1.8\ \text{s}) \approx 39\ \text{seats}\,.$$

5. Let $y_1$ = 2.0 mm (corresponding to time $t_1$) and $y_2$ = –2.0 mm (corresponding to time $t_2$). Then we find

$$kx + 600t_1 + \phi = \sin^{-1}(2.0/6.0)$$

and

$$kx + 600t_2 + \phi = \sin^{-1}(-2.0/6.0) \ .$$

Subtracting equations gives

$$600(t_1 - t_2) = \sin^{-1}(2.0/6.0) - \sin^{-1}(-2.0/6.0).$$

Thus we find $t_1 - t_2$ = 0.011 s (or 1.1 ms).

6. Setting $x = 0$ in $u = -\omega y_m \cos(kx - \omega t + \phi)$ (see Eq. 16-21 or Eq. 16-28) gives

$$u = -\omega y_m \cos(-\omega t + \phi)$$

as the function being plotted in the graph. We note that it has a positive "slope" (referring to its $t$-derivative) at $t = 0$:

$$\frac{du}{dt} = \frac{d(-\omega\, y_m \cos(-\omega t + \phi))}{dt} = -y_m \omega^2 \sin(-\omega t + \phi) \quad > 0 \ \text{ at } \ t = 0.$$

This implies that $-\sin\phi > 0$ and consequently that $\phi$ is in either the third or fourth quadrant. The graph shows (at t = 0) $u = -4$ m/s, and (at some later $t$) $u_{max} = 5$ m/s. We note that $u_{max} = y_m \omega$. Therefore,

$$u = -u_{max} \cos(-\omega t + \phi)\Big|_{t=0} \quad \Rightarrow \quad \phi = \cos^{-1}\left(\tfrac{4}{5}\right) = \pm 0.6435 \text{ rad}$$

(bear in mind that $\cos\theta = \cos(-\theta)$), and we must choose $\phi = -0.64$ rad (since this is about $-37°$ and is in fourth quadrant). Of course, this answer added to $2n\pi$ is still a valid answer (where n is any integer), so that, for example, $\phi = -0.64 + 2\pi = 5.64$ rad is also an acceptable result.

7. Using $v = f\lambda$, we find the length of one cycle of the wave is

$$\lambda = 350/500 = 0.700 \text{ m} = 700 \text{ mm}.$$

From $f = 1/T$, we find the time for one cycle of oscillation is $T = 1/500 = 2.00 \times 10^{-3}$ s = 2.00 ms.

(a) A cycle is equivalent to $2\pi$ radians, so that $\pi/3$ rad corresponds to one-sixth of a cycle. The corresponding length, therefore, is $\lambda/6 = 700/6 = 117$ mm.

(b) The interval 1.00 ms is half of $T$ and thus corresponds to half of one cycle, or half of $2\pi$ rad. Thus, the phase difference is $(1/2)2\pi = \pi$ rad.

8. (a) The amplitude is $y_m = 6.0$ cm.

(b) We find $\lambda$ from $2\pi/\lambda = 0.020\pi$: $\lambda = 1.0\times10^2$ cm.

(c) Solving $2\pi f = \omega = 4.0\pi$, we obtain $f = 2.0$ Hz.

(d) The wave speed is $v = \lambda f = (100 \text{ cm})(2.0 \text{ Hz}) = 2.0\times10^2$ cm/s.

(e) The wave propagates in the $-x$ direction, since the argument of the trig function is $kx + \omega t$ instead of $kx - \omega t$ (as in Eq. 16-2).

(f) The maximum transverse speed (found from the time derivative of $y$) is

$$u_{\max} = 2\pi f y_m = \left(4.0\pi \text{ s}^{-1}\right)\left(6.0 \text{ cm}\right) = 75 \text{ cm/s}.$$

(g) $y(3.5 \text{ cm}, 0.26 \text{ s}) = (6.0 \text{ cm}) \sin[0.020\pi(3.5) + 4.0\pi(0.26)] = -2.0$ cm.

9. (a) Recalling from Ch. 12 the simple harmonic motion relation $u_m = y_m\omega$, we have

$$\omega = \frac{16}{0.040} = 400\,\text{rad/s}.$$

Since $\omega = 2\pi f$, we obtain $f$ = 64 Hz.

(b) Using $v = f\lambda$, we find $\lambda$ = 80/64 = 1.26 m $\approx 1.3$ m.

(c) The amplitude of the transverse displacement is $y_m = 4.0\text{ cm} = 4.0\times10^{-2}\,\text{m}$.

(d) The wave number is $k = 2\pi/\lambda$ = 5.0 rad/m.

(e) The angular frequency, as obtained in part (a), is $\omega = 16/0.040 = 4.0\times10^{2}$ rad/s.

(f) The function describing the wave can be written as

$$y = 0.040\sin(5x - 400t + \phi)$$

where distances are in meters and time is in seconds. We adjust the phase constant $\phi$ to satisfy the condition $y$ = 0.040 at $x = t = 0$. Therefore, sin $\phi$ = 1, for which the "simplest" root is $\phi = \pi/2$. Consequently, the answer is

$$y = 0.040\sin\left(5x - 400t + \frac{\pi}{2}\right).$$

(g) The sign in front of $\omega$ is minus.

10. With length in centimeters and time in seconds, we have

$$u = \frac{du}{dt} = 225\pi \sin(\pi x - 15\pi t) .$$

Squaring this and adding it to the square of $15\pi y$, we have

$$u^2 + (15\pi y)^2 = (225\pi)^2 [\sin^2 (\pi x - 15\pi t) + \cos^2 (\pi x - 15\pi t)]$$

so that

$$u = \sqrt{(225\pi)^2 - (15\pi y)^2} = 15\pi \sqrt{15^2 - y^2} .$$

Therefore, where $y = 12$, $u$ must be $\pm 135\pi$. Consequently, the *speed* there is 424 cm/s = 4.24 m/s.

11. (a) The amplitude $y_m$ is half of the 6.00 mm vertical range shown in the figure, i.e., $y_m = 3.0$ mm.

(b) The speed of the wave is $v = d/t$ = 15 m/s, where $d$ = 0.060 m and $t$ = 0.0040 s. The angular wave number is $k = 2\pi/\lambda$ where $\lambda$ = 0.40 m. Thus,

$$k = \frac{2\pi}{\lambda} = 16 \text{ rad/m} .$$

(c) The angular frequency is found from

$$\omega = k v = (16 \text{ rad/m})(15 \text{ m/s}) = 2.4 \times 10^2 \text{ rad/s}.$$

(d) We choose the minus sign (between $kx$ and $\omega t$) in the argument of the sine function because the wave is shown traveling to the right [in the $+x$ direction] – see section 16-5). Therefore, with SI units understood, we obtain

$$y = y_m \sin(kx - kvt) \approx 0.0030 \sin(16\,x - 2.4 \times 10^2\, t) .$$

12. The slope that they are plotting is the physical slope of sinusoidal waveshape (not to be confused with the more abstract “slope” of its time development; the physical slope is an *x*-derivative whereas the more abstract “slope” would be the *t*-derivative). Thus, where the figure shows a maximum slope equal to 0.2 (with no unit), it refers to the maximum of the following function:

$$\frac{dy}{dx} = \frac{d\, y_m \sin(kx - \omega t)}{dx} = y_m\, k \cos(kx - \omega t)\ .$$

The problem additionally gives $t = 0$, which we can substitute into the above expression if desired. In any case, the maximum of the above expression is $y_m k$, where

$$k = \frac{2\pi}{\lambda} = \frac{2\pi}{0.40\text{ m}} = 15.7\text{ rad/m}.$$

Therefore, setting $y_m k$ equal to 0.20 allows us to solve for the amplitude $y_m$. We find

$$y_m = \frac{0.20}{15.7\text{ rad/m}} = 0.0127\text{ m} \approx 1.3\text{ cm}.$$

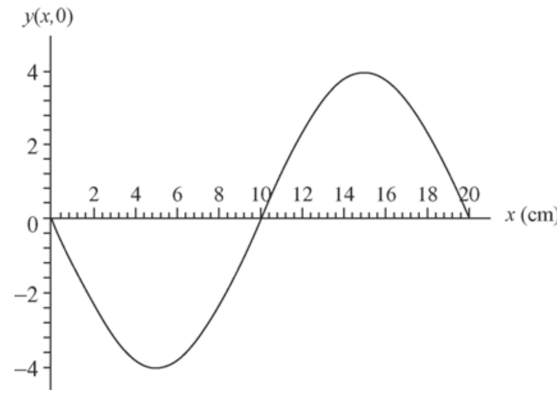
13. From Eq. 16-10, a general expression for a sinusoidal wave traveling along the $+x$ direction is

$$y(x,t) = y_m \sin(kx - \omega t + \phi)$$

(a) The figure shows that at $x = 0$, $y(0,t) = y_m \sin(-\omega t + \phi)$ is a positive sine function, i.e., $y(0,t) = +y_m \sin \omega t$. Therefore, the phase constant must be $\phi = \pi$. At $t$ =0, we then have

$$y(x,0) = y_m \sin(kx + \pi) = -y_m \sin kx$$

which is a negative sine function. A plot of $y$(x,0) is depicted on the right.




(b) From the figure we see that the amplitude is $y_m$ = 4.0 cm.

(c) The angular wave number is given by $k = 2\pi/\lambda = \pi/10 = 0.31$ rad/cm.

(d) The angular frequency is $\omega = 2\pi/T = \pi/5 = 0.63$ rad/s.

(e) As found in part (a), the phase is $\phi = \pi$.

(f) The sign is minus since the wave is traveling in the $+x$ direction.

(g) Since the frequency is $f = 1/T = 0.10$ s, the speed of the wave is $v = f\lambda = 2.0$ cm/s.

(h) From the results above, the wave may be expressed as

$$y(x,t) = 4.0\sin\left(\frac{\pi x}{10} - \frac{\pi t}{5} + \pi\right) = -4.0\sin\left(\frac{\pi x}{10} - \frac{\pi t}{5}\right).$$

Taking the derivative of $y$ with respect to $t$, we find

$$u(x,t) = \frac{\partial y}{\partial t} = 4.0\left(\frac{\pi}{t}\right)\cos\left(\frac{\pi x}{10} - \frac{\pi t}{5}\right)$$

which yields $u(0,5.0) = -2.5$ cm/s.

14. From $v = \sqrt{\tau/\mu}$ , we have

$$\frac{v_{\text{new}}}{v_{\text{old}}} = \frac{\sqrt{\tau_{\text{new}}/\mu_{\text{new}}}}{\sqrt{\tau_{\text{old}}/\mu_{\text{old}}}} = \sqrt{2}.$$

15. The wave speed $v$ is given by $v = \sqrt{\tau/\mu}$, where $\tau$ is the tension in the rope and $\mu$ is the linear mass density of the rope. The linear mass density is the mass per unit length of rope:

$$\mu = m/L = (0.0600 \text{ kg})/(2.00 \text{ m}) = 0.0300 \text{ kg/m}.$$

Thus,

$$v = \sqrt{\frac{500\,\text{N}}{0.0300\,\text{kg/m}}} = 129\,\text{m/s}.$$

16. The volume of a cylinder of height $\ell$ is $V = \pi r^2 \ell = \pi d^2 \ell/4$. The strings are long, narrow cylinders, one of diameter $d_1$ and the other of diameter $d_2$ (and corresponding linear densities $\mu_1$ and $\mu_2$). The mass is the (regular) density multiplied by the volume: $m = \rho V$, so that the mass-per-unit length is

$$\mu = \frac{m}{\ell} = \frac{\rho \pi d^2 \ell/4}{\ell} = \frac{\pi \rho d^2}{4}$$

and their ratio is

$$\frac{\mu_1}{\mu_2} = \frac{\pi\rho\, d_1^2/4}{\pi\rho\, d_2^2/4} = \left(\frac{d_1}{d_2}\right)^2 .$$

Therefore, the ratio of diameters is

$$\frac{d_1}{d_2} = \sqrt{\frac{\mu_1}{\mu_2}} = \sqrt{\frac{3.0}{0.29}} = 3.2.$$

17. (a) The amplitude of the wave is $y_m$=0.120 mm.

(b) The wave speed is given by $v = \sqrt{\tau/\mu}$, where $\tau$ is the tension in the string and $\mu$ is the linear mass density of the string, so the wavelength is $\lambda = v/f = \sqrt{\tau/\mu}/f$ and the angular wave number is

$$k = \frac{2\pi}{\lambda} = 2\pi f\sqrt{\frac{\mu}{\tau}} = 2\pi(100\,\text{Hz})\sqrt{\frac{0.50\,\text{kg/m}}{10\,\text{N}}} = 141\,\text{m}^{-1}.$$

(c) The frequency is $f$ = 100 Hz, so the angular frequency is

$$\omega = 2\pi f = 2\pi(100\text{ Hz}) = 628\text{ rad/s}.$$

(d) We may write the string displacement in the form $y = y_m \sin(kx + \omega t)$. The plus sign is used since the wave is traveling in the negative $x$ direction. In summary, the wave can be expressed as

$$y = (0.120\,\text{mm})\sin\left[(141\,\text{m}^{-1})x + (628\,\text{s}^{-1})t\right].$$

18. We use $v=\sqrt{\tau/\mu}\propto\sqrt{\tau}$ to obtain

$$\tau_2=\tau_1\left(\frac{v_2}{v_1}\right)^2=(120\,\mathrm{N})\left(\frac{180\,\mathrm{m/s}}{170\,\mathrm{m/s}}\right)^2=135\mathrm{N}.$$

19. (a) The wave speed is given by $v = \lambda/T = \omega/k$, where $\lambda$ is the wavelength, $T$ is the period, $\omega$ is the angular frequency ($2\pi/T$), and $k$ is the angular wave number ($2\pi/\lambda$). The displacement has the form $y = y_m \sin(kx + \omega t)$, so $k = 2.0\ \text{m}^{-1}$ and $\omega = 30$ rad/s. Thus

$$v = (30\ \text{rad/s})/(2.0\ \text{m}^{-1}) = 15\ \text{m/s}.$$

(b) Since the wave speed is given by $v = \sqrt{\tau/\mu}$, where $\tau$ is the tension in the string and $\mu$ is the linear mass density of the string, the tension is

$$\tau = \mu v^2 = \left(1.6\times10^{-4}\ \text{kg/m}\right)\left(15\,\text{m/s}\right)^2 = 0.036\,\text{N}.$$

20. (a) Comparing with Eq. 16-2, we see that $k$ = 20/m and $\omega$ = 600/s. Therefore, the speed of the wave is (see Eq. 16-13) $v = \omega/k$ = 30 m/s.

(b) From Eq. 16–26, we find

$$\mu = \frac{\tau}{v^2} = \frac{15}{30^2} = 0.017\,\text{kg/m} = 17\,\text{g/m}.$$

21. (a) We read the amplitude from the graph. It is about 5.0 cm.

(b) We read the wavelength from the graph. The curve crosses $y = 0$ at about $x = 15$ cm and again with the same slope at about $x = 55$ cm, so

$$\lambda = (55 \text{ cm} - 15 \text{ cm}) = 40 \text{ cm} = 0.40 \text{ m}.$$

(c) The wave speed is $v = \sqrt{\tau / \mu}$, where $\tau$ is the tension in the string and $\mu$ is the linear mass density of the string. Thus,

$$v = \sqrt{\frac{3.6 \text{ N}}{25 \times 10^{-3} \text{ kg/m}}} = 12 \text{ m/s}.$$

(d) The frequency is $f = v/\lambda = (12 \text{ m/s})/(0.40 \text{ m}) = 30$ Hz and the period is

$$T = 1/f = 1/(30 \text{ Hz}) = 0.033 \text{ s}.$$

(e) The maximum string speed is

$$u_m = \omega y_m = 2\pi f y_m = 2\pi(30 \text{ Hz})\,(5.0 \text{ cm}) = 940 \text{ cm/s} = 9.4 \text{ m/s}.$$

(f) The angular wave number is $k = 2\pi/\lambda = 2\pi/(0.40 \text{ m}) = 16 \text{ m}^{-1}$.

(g) The angular frequency is $\omega = 2\pi f = 2\pi(30 \text{ Hz}) = 1.9\times10^2$ rad/s

(h) According to the graph, the displacement at $x = 0$ and $t = 0$ is $4.0 \times 10^{-2}$ m. The formula for the displacement gives $y(0, 0) = y_m \sin \phi$. We wish to select $\phi$ so that

$$5.0 \times 10^{-2} \sin \phi = 4.0 \times 10^{-2}.$$

The solution is either 0.93 rad or 2.21 rad. In the first case the function has a positive slope at $x = 0$ and matches the graph. In the second case it has negative slope and does not match the graph. We select $\phi = 0.93$ rad.

(i) The string displacement has the form $y\,(x, t) = y_m \sin(kx + \omega t + \phi)$. A plus sign appears in the argument of the trigonometric function because the wave is moving in the negative $x$ direction. Using the results obtained above, the expression for the displacement is

$$y(x,t) = \left(5.0\times10^{-2} \text{ m}\right)\sin\left[(16 \text{ m}^{-1})x + (190 \text{ s}^{-1})t + 0.93\right].$$

22. (a) The general expression for $y(x, t)$ for the wave is $y(x, t) = y_m \sin(kx - \omega t)$, which, at $x$ = 10 cm, becomes $y(x = 10\text{ cm}, t) = y_m \sin[k(10\text{ cm} - \omega t)]$. Comparing this with the expression given, we find $\omega$ = 4.0 rad/s, or $f = \omega/2\pi$ = 0.64 Hz.

(b) Since $k$(10 cm) = 1.0, the wave number is $k$ = 0.10/cm. Consequently, the wavelength is $\lambda = 2\pi/k$ = 63 cm.

(c) The amplitude is $y_m = 5.0$ cm.

(d) In part (b), we have shown that the angular wave number is $k$ = 0.10/cm.

(e) The angular frequency is $\omega$ = 4.0 rad/s.

(f) The sign is minus since the wave is traveling in the $+x$ direction.

Summarizing the results obtained above by substituting the values of $k$ and $\omega$ into the general expression for $y(x, t)$, with centimeters and seconds understood, we obtain

$$y(x,t) = 5.0\sin(0.10x - 4.0t).$$

(g) Since $v = \omega/k = \sqrt{\tau/\mu}$, the tension is

$$\tau = \frac{\omega^2\mu}{k^2} = \frac{(4.0\,\text{g/cm})(4.0\,\text{s}^{-1})^2}{(0.10\,\text{cm}^{-1})^2} = 6400\,\text{g}\cdot\text{cm/s}^2 = 0.064\,\text{N}.$$

23. The pulses have the same speed $v$. Suppose one pulse starts from the left end of the wire at time $t = 0$. Its coordinate at time $t$ is $x_1 = vt$. The other pulse starts from the right end, at $x = L$, where $L$ is the length of the wire, at time $t = 30$ ms. If this time is denoted by $t_0$ then the coordinate of this wave at time $t$ is $x_2 = L - v(t - t_0)$. They meet when $x_1 = x_2$, or, what is the same, when $vt = L - v(t - t_0)$. We solve for the time they meet: $t = (L + vt_0)/2v$ and the coordinate of the meeting point is $x = vt = (L + vt_0)/2$. Now, we calculate the wave speed:

$$v = \sqrt{\frac{\tau L}{m}} = \sqrt{\frac{(250\,\text{N})(10.0\,\text{m})}{0.100\,\text{kg}}} = 158\,\text{m/s}.$$

Here $\tau$ is the tension in the wire and $L/m$ is the linear mass density of the wire. The coordinate of the meeting point is

$$x = \frac{10.0\,\text{m} + (158\,\text{m/s})(30.0\times10^{-3}\,\text{s})}{2} = 7.37\,\text{m}.$$

This is the distance from the left end of the wire. The distance from the right end is $L - x = (10.0\text{ m} - 7.37\text{ m}) = 2.63$ m.

24. (a) The tension in each string is given by $\tau = Mg/2$. Thus, the wave speed in string 1 is

$$v_1 = \sqrt{\frac{\tau}{\mu_1}} = \sqrt{\frac{Mg}{2\mu_1}} = \sqrt{\frac{(500\,\text{g})(9.80\,\text{m/s}^2)}{2(3.00\,\text{g/m})}} = 28.6\,\text{m/s}.$$

(b) And the wave speed in string 2 is

$$v_2 = \sqrt{\frac{Mg}{2\mu_2}} = \sqrt{\frac{(500\,\text{g})(9.80\,\text{m/s}^2)}{2(5.00\,\text{g/m})}} = 22.1\,\text{m/s}.$$

(c) Let $v_1 = \sqrt{M_1 g/(2\mu_1)} = v_2 = \sqrt{M_2 g/(2\mu_2)}$ and $M_1 + M_2 = M$. We solve for $M_1$ and obtain

$$M_1 = \frac{M}{1+\mu_2/\mu_1} = \frac{500\,\text{g}}{1+5.00/3.00} = 187.5\,\text{g} \approx 188\,\text{g}.$$

(d) And we solve for the second mass: $M_2 = M - M_1 = (500\text{ g} - 187.5\text{ g}) \approx 313\text{ g}$.

25. (a) The wave speed at any point on the rope is given by $v = \sqrt{\tau/\mu}$, where $\tau$ is the tension at that point and $\mu$ is the linear mass density. Because the rope is hanging the tension varies from point to point. Consider a point on the rope a distance *y* from the bottom end. The forces acting on it are the weight of the rope below it, pulling down, and the tension, pulling up. Since the rope is in equilibrium, these forces balance. The weight of the rope below is given by $\mu g y$, so the tension is $\tau = \mu g y$. The wave speed is $v = \sqrt{\mu g y/\mu} = \sqrt{gy}.$

(b) The time d*t* for the wave to move past a length d*y*, a distance *y* from the bottom end, is $dt = dy/v = dy\big/\sqrt{gy}$ and the total time for the wave to move the entire length of the rope is

$$t = \int_0^L \frac{dy}{\sqrt{gy}} = 2\sqrt{\frac{y}{g}}\,\Bigg|_0^L = 2\sqrt{\frac{L}{g}}\,.$$

26. Using Eq. 16–33 for the average power and Eq. 16–26 for the speed of the wave, we solve for $f = \omega/2\pi$:

$$f = \frac{1}{2\pi y_m}\sqrt{\frac{2P_{\text{avg}}}{\mu\sqrt{\tau/\mu}}} = \frac{1}{2\pi(7.70\times10^{-3}\,\text{m})}\sqrt{\frac{2(85.0\,\text{W})}{\sqrt{(36.0\,\text{N})(0.260\,\text{kg}/2.70\,\text{m})}}} = 198\ \text{Hz}.$$

27. We note from the graph (and from the fact that we are dealing with a cosine-squared, see Eq. 16-30) that the wave frequency is $f = \frac{1}{2\text{ ms}} = 500$ Hz, and that the wavelength $\lambda$ = 0.20 m. We also note from the graph that the maximum value of *dK/dt* is 10 W. Setting this equal to the maximum value of Eq. 16-29 (where we just set that cosine term equal to 1) we find

$$\frac{1}{2}\mu v\, \omega^2 y_m^{\ 2} = 10$$

with SI units understood. Substituting in $\mu = 0.002$ kg/m, $\omega = 2\pi f$ and $v = f\lambda$, we solve for the wave amplitude:

$$y_m = \sqrt{\frac{10}{2\pi^2 \mu\lambda\ f^3}} = 0.0032 \text{ m} .$$

28. Comparing $y(x,t) = (3.00 \text{ mm})\sin[(4.00 \text{ m}^{-1})x - (7.00 \text{ s}^{-1})t]$ to the general expression $y(x,t) = y_m \sin(kx - \omega t)$, we see that $k = 4.00 \text{ m}^{-1}$ and $\omega = 7.00 \text{ rad/s}$. The speed of the wave is

$$v = \omega / k = (7.00 \text{ rad/s})/(4.00 \text{ m}^{-1}) = 1.75 \text{ m/s}.$$

29. The wave $y(x,t) = (2.00 \text{ mm})[(20 \text{ m}^{-1})x - (4.0 \text{ s}^{-1})t]^{1/2}$ is of the form $h(kx - \omega t)$ with angular wave number $k = 20 \text{ m}^{-1}$ and angular frequency $\omega = 4.0 \text{ rad/s}$. Thus, the speed of the wave is

$$v = \omega / k = (4.0 \text{ rad/s})/(20 \text{ m}^{-1}) = 0.20 \text{ m/s}.$$

30. The wave $y(x,t) = (4.00 \text{ mm})\, h[(30 \text{ m}^{-1})x + (6.0 \text{ s}^{-1})t]$ is of the form $h(kx - \omega t)$ with angular wave number $k = 30 \text{ m}^{-1}$ and angular frequency $\omega = 6.0 \text{ rad/s}$. Thus, the speed of the wave is

$$v = \omega / k = (6.0 \text{ rad/s})/(30 \text{ m}^{-1}) = 0.20 \text{ m/s}.$$

31. The displacement of the string is given by

$$y = y_m \sin(kx - \omega t) + y_m \sin(kx - \omega t + \phi) \ = 2y_m \cos\left(\tfrac{1}{2}\phi\right)\sin\left(kx - \omega t + \tfrac{1}{2}\phi\right),$$

where $\phi = \pi/2$. The amplitude is

$$A = 2y_m \cos\left(\tfrac{1}{2}\phi\right) = 2y_m \cos(\pi/4) = 1.41 y_m .$$

32. (a) Let the phase difference be $\phi$. Then from Eq. 16–52, $2y_m \cos(\phi/2) = 1.50y_m$, which gives

$$\phi = 2\cos^{-1}\left(\frac{1.50y_m}{2y_m}\right) = 82.8°.$$

(b) Converting to radians, we have $\phi = 1.45$ rad.

(c) In terms of wavelength (the length of each cycle, where each cycle corresponds to $2\pi$ rad), this is equivalent to $1.45 \text{ rad}/2\pi = 0.230$ wavelength.

33. (a) The amplitude of the second wave is $y_m = 9.00$ mm, as stated in the problem.

(b) The figure indicates that λ = 40 cm = 0.40 m, which implies that the angular wave number is $k = 2\pi/0.40 = 16$ rad/m.

(c) The figure (along with information in the problem) indicates that the speed of each wave is $v = dx/t$ = (56.0 cm)/(8.0 ms) = 70 m/s. This, in turn, implies that the angular frequency is

$$\omega = kv = 1100 \text{ rad/s} = 1.1\times10^3 \text{ rad/s}.$$

(d) The figure depicts two traveling waves (both going in the $-x$ direction) of equal amplitude $y_m$. The amplitude of their resultant wave, as shown in the figure, is $y'_m$ = 4.00 mm. Eq. 16-52 applies:

$$y'_m = 2\,y_m \cos(\tfrac{1}{2}\phi_2) \quad \Rightarrow \quad \phi_2 = 2\cos^{-1}(2.00/9.00) = 2.69 \text{ rad}.$$

(e) In making the plus-or-minus sign choice in $y = y_m \sin(kx \pm \omega t + \phi)$, we recall the discussion in section 16-5, where it shown that sinusoidal waves traveling in the $-x$ direction are of the form $y = y_m \sin(kx + \omega t + \phi)$. Here, ϕ should be thought of as the phase *difference* between the two waves (that is, $\phi_1 = 0$ for wave 1 and $\phi_2 = 2.69$ rad for wave 2).

In summary, the waves have the forms (with SI units understood):

$$y_1 = (0.00900)\sin(16x + 1100t) \quad \text{and} \quad y_2 = (0.00900)\sin(16x + 1100t + 2.7)\ .$$

34. (a) We use Eq. 16-26 and Eq. 16-33 with $\mu$ = 0.00200 kg/m and $y_m$ = 0.00300 m. These give $v = \sqrt{\tau / \mu}$ = 775 m/s and

$$P_{\text{avg}} = \tfrac{1}{2}\,\mu v\,\omega^2 y_m^{\,2} = 10\text{ W}.$$

(b) In this situation, the waves are two separate string (no superposition occurs). The answer is clearly twice that of part (a); $P$ = 20 W.

(c) Now they are on the same string. If they are interfering constructively (as in Fig. 16-16(a)) then the amplitude $y_m$ is doubled which means its square $y_m^{\,2}$ increases by a factor of 4. Thus, the answer now is four times that of part (a); $P$ = 40 W.

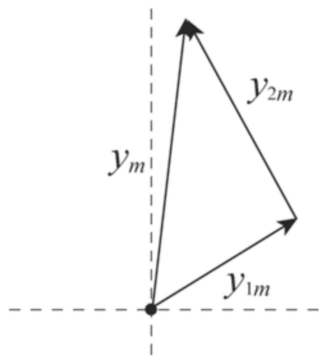
(d) Eq. 16-52 indicates in this case that the amplitude (for their superposition) is $2\,y_m\cos(0.2\pi)$ = 1.618 times the original amplitude $y_m$. Squared, this results in an increase in the power by a factor of 2.618. Thus, $P$ = 26 W in this case.

(e) Now the situation depicted in Fig. 16-16(b) applies, so $P$ = 0.

35. The phasor diagram is shown below: $y_{1m}$ and $y_{2m}$ represent the original waves and $y_m$ represents the resultant wave. The phasors corresponding to the two constituent waves make an angle of 90° with each other, so the triangle is a right triangle. The Pythagorean theorem gives

$$y_m^2 = y_{1m}^2 + y_{2m}^2 = (3.0\,\text{cm})^2 + (4.0\,\text{cm})^2 = (25\,\text{cm})^2 .$$

Thus $y_m$ = 5.0 cm.



36. (a) As shown in Figure 16-16(b) in the textbook, the least-amplitude resultant wave is obtained when the phase difference is π rad.

(b) In this case, the amplitude is (8.0 mm – 5.0 mm) = 3.0 mm.

(c) As shown in Figure 16-16(a) in the textbook, the greatest-amplitude resultant wave is obtained when the phase difference is 0 rad.

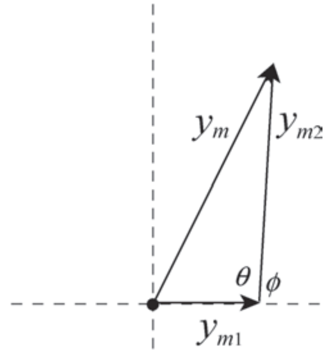
(d) In the part (c) situation, the amplitude is (8.0 mm + 5.0 mm) = 13 mm.

(e) Using phasor terminology, the angle "between them" in this case is π/2 rad (90º), so the Pythagorean theorem applies:

$$\sqrt{(8.0\text{ mm})^2 + (5.0\text{ mm})^2} = 9.4\text{ mm}.$$

37. The phasor diagram is shown on the right. We use the cosine theorem:

$$y_m^2 = y_{m1}^2 + y_{m2}^2 - 2y_{m1}y_{m2}\cos\theta = y_{m1}^2 + y_{m2}^2 + 2y_{m1}y_{m2}\cos\phi.$$




We solve for cos $\phi$:

$$\cos\phi = \frac{y_m^2 - y_{m1}^2 - y_{m2}^2}{2y_{m1}\,y_{m2}} = \frac{(9.0\,\text{mm})^2 - (5.0\,\text{mm})^2 - (7.0\,\text{mm})^2}{2(5.0\,\text{mm})(7.0\,\text{mm})} = 0.10.$$

The phase constant is therefore $\phi = 84°$.

38. We see that $y_1$ and $y_3$ cancel (they are 180º) out of phase, and $y_2$ cancels with $y_4$ because their phase difference is also equal to π rad (180º). There is no resultant wave in this case.

39. (a) Using the phasor technique, we think of these as two “vectors” (the first of “length” 4.6 mm and the second of “length” 5.60 mm) separated by an angle of $\phi = 0.8\pi$ radians (or 144º). Standard techniques for adding vectors then lead to a resultant vector of length 3.29 mm.

(b) The angle (relative to the first vector) is equal to 88.8º (or 1.55 rad).

(c) Clearly, it should in “in phase” with the result we just calculated, so its phase angle relative to the first phasor should be also 88.8º (or 1.55 rad).

40. (a) The wave speed is given by

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{7.00\,\text{N}}{2.00 \times 10^{-3}\,\text{kg}/1.25\,\text{m}}} = 66.1\,\text{m/s}.$$

(b) The wavelength of the wave with the lowest resonant frequency $f_1$ is $\lambda_1 = 2L$, where $L$ = 125 cm. Thus,

$$f_1 = \frac{v}{\lambda_1} = \frac{66.1\text{ m/s}}{2(1.25\text{ m})} = 26.4\text{ Hz}.$$

41. Possible wavelengths are given by $\lambda = 2L/n$, where $L$ is the length of the wire and $n$ is an integer. The corresponding frequencies are given by $f = v/\lambda = nv/2L$, where $v$ is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu} = \sqrt{\tau L / M}$, where $\tau$ is the tension in the wire, $\mu$ is the linear mass density of the wire, and $M$ is the mass of the wire. $\mu = M/L$ was used to obtain the last form. Thus

$$f_n = \frac{n}{2L}\sqrt{\frac{\tau L}{M}} = \frac{n}{2}\sqrt{\frac{\tau}{LM}} = \frac{n}{2}\sqrt{\frac{250\text{ N}}{(10.0\text{ m})\,(0.100\text{ kg})}} = n\ (7.91\text{ Hz}).$$

(a) The lowest frequency is $f_1 = 7.91$ Hz.

(b) The second lowest frequency is $f_2 = 2(7.91\text{ Hz}) = 15.8$ Hz.

(c) The third lowest frequency is $f_3 = 3(7.91\text{ Hz}) = 23.7$ Hz.

42. The *n*th resonant frequency of string *A* is

$$f_{n,A} = \frac{v_A}{2l_A} n = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}},$$

while for string *B* it is

$$f_{n,B} = \frac{v_B}{2l_B} n = \frac{n}{8L}\sqrt{\frac{\tau}{\mu}} = \frac{1}{4} f_{n,A}.$$

(a) Thus, we see $f_{1,A} = f_{4,B}$. That is, the fourth harmonic of *B* matches the frequency of *A*'s first harmonic.

(b) Similarly, we find $f_{2,A} = f_{8,B}$.

(c) No harmonic of *B* would match $f_{3,A} = \frac{3v_A}{2l_A} = \frac{3}{2L}\sqrt{\frac{\tau}{\mu}},$

43. (a) The wave speed is given by $v = \sqrt{\tau/\mu}$, where $\tau$ is the tension in the string and $\mu$ is the linear mass density of the string. Since the mass density is the mass per unit length, $\mu = M/L$, where $M$ is the mass of the string and $L$ is its length. Thus

$$v = \sqrt{\frac{\tau L}{M}} = \sqrt{\frac{(96.0\ \text{N})\ (8.40\ \text{m})}{0.120\ \text{kg}}} = 82.0\ \text{m/s}.$$

(b) The longest possible wavelength $\lambda$ for a standing wave is related to the length of the string by $L = \lambda/2$, so $\lambda = 2L = 2(8.40\ \text{m}) = 16.8\ \text{m}$.

(c) The frequency is $f = v/\lambda = (82.0\ \text{m/s})/(16.8\ \text{m}) = 4.88\ \text{Hz}$.

44. The string is flat each time the particle passes through its equilibrium position. A particle may travel up to its positive amplitude point and back to equilibrium during this time. This describes *half* of one complete cycle, so we conclude $T$ = 2(0.50 s) = 1.0 s. Thus, $f = 1/T$ = 1.0 Hz, and the wavelength is

$$\lambda = \frac{v}{f} = \frac{10\,\text{cm/s}}{1.0\,\text{Hz}} = 10\ \text{cm.}$$

45. (a) Eq. 16–26 gives the speed of the wave:

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{150\,\text{N}}{7.20\times10^{-3}\ \text{kg/m}}} = 144.34\ \text{m/s} \approx 1.44\times10^{2}\ \text{m/s}.$$

(b) From the figure, we find the wavelength of the standing wave to be

$$\lambda = (2/3)(90.0\ \text{cm}) = 60.0\ \text{cm}.$$

(c) The frequency is

$$f = \frac{v}{\lambda} = \frac{1.44\times10^{2}\ \text{m/s}}{0.600\,\text{m}} = 241\,\text{Hz}.$$

46. Use Eq. 16–66 (for the resonant frequencies) and Eq. 16–26 $(v=\sqrt{\tau/\mu})$ to find $f_n$:

$$f_n=\frac{nv}{2L}=\frac{n}{2L}\sqrt{\frac{\tau}{\mu}}$$

which gives $f_3 = (3/2L)\sqrt{\tau_i/\mu}$ .

(a) When $\tau_f = 4\tau_i$, we get the new frequency

$$f_3'=\frac{3}{2L}\sqrt{\frac{\tau_f}{\mu}}=2f_3.$$

(b) And we get the new wavelength $\lambda_3'=\dfrac{v'}{f_3'}=\dfrac{2L}{3}=\lambda_3$.

47. (a) The resonant wavelengths are given by $\lambda = 2L/n$, where $L$ is the length of the string and $n$ is an integer, and the resonant frequencies are given by $f = v/\lambda = nv/2L$, where $v$ is the wave speed. Suppose the lower frequency is associated with the integer $n$. Then, since there are no resonant frequencies between, the higher frequency is associated with $n + 1$. That is, $f_1 = nv/2L$ is the lower frequency and $f_2 = (n + 1)v/2L$ is the higher. The ratio of the frequencies is

$$\frac{f_2}{f_1} = \frac{n+1}{n}.$$

The solution for $n$ is

$$n = \frac{f_1}{f_2 - f_1} = \frac{315 \text{ Hz}}{420 \text{ Hz} - 315 \text{ Hz}} = 3.$$

The lowest possible resonant frequency is $f = v/2L = f_1/n = (315 \text{ Hz})/3 = 105$ Hz.

(b) The longest possible wavelength is $\lambda = 2L$. If $f$ is the lowest possible frequency then

$$v = \lambda f = 2Lf = 2(0.75 \text{ m})(105 \text{ Hz}) = 158 \text{ m/s}.$$

48. Using Eq. 16-26, we find the wave speed to be

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{65.2\times10^6\ \text{N}}{3.35\,\text{kg/m}}} = 4412\,\text{m/s}.$$

The corresponding resonant frequencies are

$$f_n = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}}, \qquad n = 1, 2, 3, \ldots$$

(a) The wavelength of the wave with the lowest (fundamental) resonant frequency $f_1$ is $\lambda_1 = 2L$, where $L = 347$ m. Thus,

$$f_1 = \frac{v}{\lambda_1} = \frac{4412\ \text{m/s}}{2(347\ \text{m})} = 6.36\ \text{Hz}.$$

(b) The frequency difference between successive modes is

$$\Delta f = f_n - f_{n-1} = \frac{v}{2L} = \frac{4412\ \text{m/s}}{2(347\ \text{m})} = 6.36\ \text{Hz}.$$

49. The harmonics are integer multiples of the fundamental, which implies that the difference between any successive pair of the harmonic frequencies is equal to the fundamental frequency. Thus, $f_1$ = (390 Hz – 325 Hz) = 65 Hz. This further implies that the next higher resonance above 195 Hz should be (195 Hz + 65 Hz) = 260 Hz.

50. Since the rope is fixed at both ends, then the phrase "second-harmonic standing wave pattern" describes the oscillation shown in Figure 16–23(b), where (see Eq. 16–65)

$$\lambda = L \quad \text{and} \quad f = \frac{v}{L}.$$

(a) Comparing the given function with Eq. 16-60, we obtain $k = \pi/2$ and $\omega = 12\pi$ rad/s. Since $k = 2\pi/\lambda$ then

$$\frac{2\pi}{\lambda} = \frac{\pi}{2} \Rightarrow \lambda = 4.0\,\text{m} \Rightarrow L = 4.0\,\text{m}.$$

(b) Since $\omega = 2\pi f$ then $2\pi f = 12\pi$ rad/s, which yields

$$f = 6.0\,\text{Hz} \Rightarrow v = f\lambda = 24\,\text{m/s}.$$

(c) Using Eq. 16–26, we have

$$v = \sqrt{\frac{\tau}{\mu}} \Rightarrow 24\ \text{m/s} = \sqrt{\frac{200\ \text{N}}{m/(4.0\ \text{m})}}$$

which leads to $m = 1.4$ kg.

(d) With

$$f = \frac{3v}{2L} = \frac{3(24\ \text{m/s})}{2(4.0\ \text{m})} = 9.0\,\text{Hz}$$

The period is $T = 1/f = 0.11$ s.

51. (a) The amplitude of each of the traveling waves is half the maximum displacement of the string when the standing wave is present, or 0.25 cm.

(b) Each traveling wave has an angular frequency of $\omega = 40\pi$ rad/s and an angular wave number of $k = \pi/3\ \text{cm}^{-1}$. The wave speed is

$$v = \omega/k = (40\pi\ \text{rad/s})/(\pi/3\ \text{cm}^{-1}) = 1.2\times10^{2}\ \text{cm/s}.$$

(c) The distance between nodes is half a wavelength: $d = \lambda/2 = \pi/k = \pi/(\pi/3\ \text{cm}^{-1}) = 3.0$ cm. Here $2\pi/k$ was substituted for $\lambda$.

(d) The string speed is given by $u(x, t) = \partial y/\partial t = -\omega y_m \sin(kx)\sin(\omega t)$. For the given coordinate and time,

$$u = -(40\pi\ \text{rad/s})\ (0.50\,\text{cm})\ \sin\left[\left(\frac{\pi}{3}\text{cm}^{-1}\right)(1.5\,\text{cm})\right]\sin\left[\left(40\pi\ \text{s}^{-1}\right)\left(\frac{9}{8}\ \text{s}\right)\right] = 0.$$

52. The nodes are located from vanishing of the spatial factor sin $5\pi x = 0$ for which the solutions are

$$5\pi x = 0, \pi, 2\pi, 3\pi, \ldots \quad \Rightarrow \quad x = 0, \frac{1}{5}, \frac{2}{5}, \frac{3}{5}, \cdots$$

(a) The smallest value of $x$ which corresponds to a node is $x = 0$.

(b) The second smallest value of $x$ which corresponds to a node is $x = 0.20$ m.

(c) The third smallest value of $x$ which corresponds to a node is $x = 0.40$ m.

(d) Every point (except at a node) is in simple harmonic motion of frequency $f = \omega/2\pi = 40\pi/2\pi = 20$ Hz. Therefore, the period of oscillation is $T = 1/f = 0.050$ s.

(e) Comparing the given function with Eq. 16–58 through Eq. 16–60, we obtain

$$y_1 = 0.020\sin(5\pi x - 40\pi t) \quad \text{and} \quad y_2 = 0.020\sin(5\pi x + 40\pi t)$$

for the two traveling waves. Thus, we infer from these that the speed is $v = \omega/k = 40\pi/5\pi = 8.0$ m/s.

(f) And we see the amplitude is $y_m = 0.020$ m.

(g) The derivative of the given function with respect to time is

$$u = \frac{\partial y}{\partial t} = -(0.040)(40\pi)\sin(5\pi x)\sin(40\pi t)$$

which vanishes (for all $x$) at times such as $\sin(40\pi t) = 0$. Thus,

$$40\pi t = 0, \pi, 2\pi, 3\pi, \ldots \quad \Rightarrow \quad t = 0, \frac{1}{40}, \frac{2}{40}, \frac{3}{40}, \cdots$$

Thus, the first time in which all points on the string have zero transverse velocity is when $t = 0$ s.

(h) The second time in which all points on the string have zero transverse velocity is when $t = 1/40$ s $= 0.025$ s.

(i) The third time in which all points on the string have zero transverse velocity is when $t = 2/40$ s $= 0.050$ s.

53. (a) The waves have the same amplitude, the same angular frequency, and the same angular wave number, but they travel in opposite directions. We take them to be

$$y_1 = y_m \sin(kx - \omega t), \quad y_2 = y_m \sin(kx + \omega t).$$

The amplitude $y_m$ is half the maximum displacement of the standing wave, or $5.0 \times 10^{-3}$ m.

(b) Since the standing wave has three loops, the string is three half-wavelengths long: $L = 3\lambda/2$, or $\lambda = 2L/3$. With $L$ = 3.0m, $\lambda$ = 2.0 m. The angular wave number is

$$k = 2\pi/\lambda = 2\pi/(2.0 \text{ m}) = 3.1 \text{ m}^{-1}.$$

(c) If $v$ is the wave speed, then the frequency is

$$f = \frac{v}{\lambda} = \frac{3v}{2L} = \frac{3(100\,\text{m/s})}{2(3.0\,\text{m})} = 50 \text{ Hz}.$$

The angular frequency is the same as that of the standing wave, or

$$\omega = 2\pi f = 2\pi(50 \text{ Hz}) = 314 \text{ rad/s}.$$

(d) The two waves are

$$y_1 = \left(5.0\times10^{-3}\,\text{m}\right)\sin\left[\left(3.14\,\text{m}^{-1}\right)x - \left(314\,\text{s}^{-1}\right)t\right]$$

and

$$y_2 = \left(5.0\times10^{-3}\,\text{m}\right)\sin\left[\left(3.14\,\text{m}^{-1}\right)x + \left(314\,\text{s}^{-1}\right)t\right].$$

Thus, if one of the waves has the form $y(x,t) = y_m \sin(kx + \omega t)$, then the other wave must have the form $y'(x,t) = y_m \sin(kx - \omega t)$. The sign in front of $\omega$ for $y'(x,t)$ is minus.

54. From the $x = 0$ plot (and the requirement of an anti-node at $x = 0$), we infer a standing wave function of the form $y(x,t) = -(0.04)\cos(kx)\sin(\omega t)$, where $\omega = 2\pi/T = \pi$ rad/s , with length in meters and time in seconds. The parameter $k$ is determined by the existence of the node at $x = 0.10$ (presumably the *first* node that one encounters as one moves from the origin in the positive $x$ direction). This implies $k(0.10) = \pi/2$ so that $k = 5\pi$ rad/m.

(a) With the parameters determined as discussed above and $t = 0.50$ s, we find

$$y(0.20 \text{ m}, 0.50 \text{ s}) = -0.04\cos(kx)\sin(\omega t) = 0.040 \text{ m} .$$

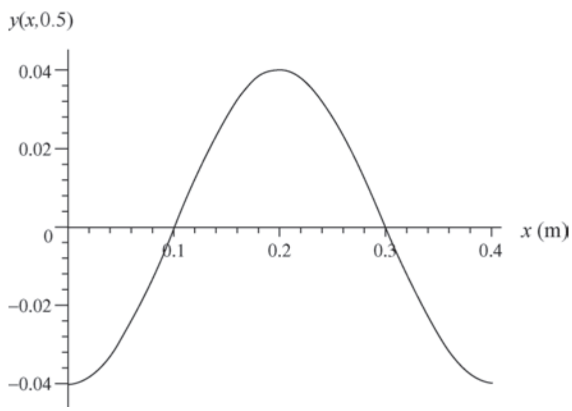
(b) The above equation yields $y(0.30 \text{ m}, 0.50 \text{ s}) = -0.04\cos(kx)\sin(\omega t) = 0$ .

(c) We take the derivative with respect to time and obtain, at $t = 0.50$ s and $x = 0.20$ m,

$$u = \frac{dy}{dt} = -0.04\omega \cos(kx)\cos(\omega t) = 0 .$$

d) The above equation yields $u = -0.13$ m/s at $t = 1.0$ s.

(e) The sketch of this function at $t = 0.50$ s for $0 \le x \le 0.40$ m is shown below:



55. (a) The angular frequency is $\omega = 8.00\pi/2 = 4.00\pi$ rad/s, so the frequency is

$$f = \omega/2\pi = (4.00\pi \text{ rad/s})/2\pi = 2.00 \text{ Hz}.$$

(b) The angular wave number is $k = 2.00\pi/2 = 1.00\pi \text{ m}^{-1}$, so the wavelength is

$$\lambda = 2\pi/k = 2\pi/(1.00\pi \text{ m}^{-1}) = 2.00 \text{ m}.$$

(c) The wave speed is

$$v = \lambda f = (2.00\,\text{m})(2.00\,\text{Hz}) = 4.00 \text{ m/s}.$$

(d) We need to add two cosine functions. First convert them to sine functions using $\cos\alpha = \sin(\alpha + \pi/2)$, then apply

$$\cos\alpha + \cos\beta = \sin\left(\alpha + \frac{\pi}{2}\right) + \sin\left(\beta + \frac{\pi}{2}\right) = 2\sin\left(\frac{\alpha+\beta+\pi}{2}\right)\cos\left(\frac{\alpha+\beta}{2}\right)$$
$$= 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

Letting $\alpha = kx$ and $\beta = \omega t$, we find

$$y_m \cos(kx+\omega t) + y_m \cos(kx - \omega t) = 2y_m \cos(kx)\cos(\omega t).$$

Nodes occur where $\cos(kx) = 0$ or $kx = n\pi + \pi/2$, where $n$ is an integer (including zero). Since $k = 1.0\pi \text{ m}^{-1}$, this means $x = \left(n + \tfrac{1}{2}\right)(1.00\,\text{m})$. Thus, the smallest value of $x$ which corresponds to a node is $x = 0.500$ m ($n$=0).

(e) The second smallest value of $x$ which corresponds to a node is $x = 1.50$ m ($n$=1).

(f) The third smallest value of $x$ which corresponds to a node is $x = 2.50$ m ($n$=2).

(g) The displacement is a maximum where $\cos(kx) = \pm 1$. This means $kx = n\pi$, where $n$ is an integer. Thus, $x = n(1.00$ m). The smallest value of $x$ which corresponds to an anti-node (maximum) is $x = 0$ ($n$=0).

(h) The second smallest value of $x$ which corresponds to an anti-node (maximum) is $x = 1.00$ m ($n$=1).

(i) The third smallest value of $x$ which corresponds to an anti-node (maximum) is $x = 2.00$ m ($n$=2).

56. Reference to point $A$ as an anti-node suggests that this is a standing wave pattern and thus that the waves are traveling in opposite directions. Thus, we expect one of them to be of the form $y = y_m \sin(kx + \omega t)$ and the other to be of the form $y = y_m \sin(kx - \omega t)$.

(a) Using Eq. 16-60, we conclude that $y_m = \frac{1}{2}(9.0 \text{ mm}) = 4.5 \text{ mm}$, due to the fact that the amplitude of the standing wave is $\frac{1}{2}(1.80 \text{ cm}) = 0.90 \text{ cm} = 9.0 \text{ mm}$.

(b) Since one full cycle of the wave (one wavelength) is 40 cm, $k = 2\pi/\lambda \approx 16 \text{ m}^{-1}$.

(c) The problem tells us that the time of half a full period of motion is 6.0 ms, so $T = 12$ ms and Eq. 16-5 gives $\omega = 5.2 \times 10^2$ rad/s.

(d) The two waves are therefore

$$y_1(x, t) = (4.5 \text{ mm}) \sin[(16 \text{ m}^{-1})x + (520 \text{ s}^{-1})t]$$

and

$$y_2(x, t) = (4.5 \text{ mm}) \sin[(16 \text{ m}^{-1})x - (520 \text{ s}^{-1})t] .$$

If one wave has the form $y(x,t) = y_m \sin(kx + \omega t)$ as in $y_1$, then the other wave must be of the form $y'(x,t) = y_m \sin(kx - \omega t)$ as in $y_2$. Therefore, the sign in front of $\omega$ is minus.

57. Recalling the discussion in section 16-12, we observe that this problem presents us with a standing wave condition with amplitude 12 cm. The angular wave number and frequency are noted by comparing the given waves with the form $y = y_m \sin(kx \pm \omega t)$. The anti-node moves through 12 cm in simple harmonic motion, just as a mass on a vertical spring would move from its upper turning point to its lower turning point – which occurs during a half-period. Since the period $T$ is related to the angular frequency by Eq. 15-5, we have

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{4.00\,\pi} = 0.500 \text{ s} .$$

Thus, in a time of $t = \frac{1}{2}T = 0.250$ s, the wave moves a distance $\Delta x = vt$ where the speed of the wave is $v = \frac{\omega}{k} = 1.00$ m/s. Therefore, $\Delta x = (1.00 \text{ m/s})(0.250 \text{ s}) = 0.250$ m.

58. With the string fixed on both ends, using Eq. 16-66 and Eq. 16-26, the resonant frequencies can be written as

$$f = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}} = \frac{n}{2L}\sqrt{\frac{mg}{\mu}}, \qquad n = 1, 2, 3, \ldots$$

(a) The mass that allows the oscillator to set up the 4th harmonic ($n = 4$) on the string is

$$m = \left.\frac{4L^2 f^2 \mu}{n^2 g}\right|_{n=4} = \frac{4(1.20\text{ m})^2(120\text{ Hz})^2(0.00160\text{ kg/m})}{(4)^2(9.80\text{ m/s}^2)} = 0.846\text{ kg}$$

(b) If the mass of the block is $m = 1.00\text{ kg}$, the corresponding $n$ is

$$n = \sqrt{\frac{4L^2 f^2 \mu}{g}} = \sqrt{\frac{4(1.20\text{ m})^2(120\text{ Hz})^2(0.00160\text{ kg/m})}{9.80\text{ m/s}^2}} = 3.68$$

which is not an integer. Therefore, the mass cannot set up a standing wave on the string.

59. (a) The frequency of the wave is the same for both sections of the wire. The wave speed and wavelength, however, are both different in different sections. Suppose there are $n_1$ loops in the aluminum section of the wire. Then,

$$L_1 = n_1\lambda_1/2 = n_1 v_1/2f,$$

where $\lambda_1$ is the wavelength and $v_1$ is the wave speed in that section. In this consideration, we have substituted $\lambda_1 = v_1/f$, where $f$ is the frequency. Thus $f = n_1 v_1/2L_1$. A similar expression holds for the steel section: $f = n_2 v_2/2L_2$. Since the frequency is the same for the two sections, $n_1 v_1/L_1 = n_2 v_2/L_2$. Now the wave speed in the aluminum section is given by $v_1 = \sqrt{\tau/\mu_1}$, where $\mu_1$ is the linear mass density of the aluminum wire. The mass of aluminum in the wire is given by $m_1 = \rho_1 A L_1$, where $\rho_1$ is the mass density (mass per unit volume) for aluminum and $A$ is the cross-sectional area of the wire. Thus

$$\mu_1 = \rho_1 A L_1/L_1 = \rho_1 A$$

and $v_1 = \sqrt{\tau/\rho_1 A}$. A similar expression holds for the wave speed in the steel section: $v_2 = \sqrt{\tau/\rho_2 A}$. We note that the cross-sectional area and the tension are the same for the two sections. The equality of the frequencies for the two sections now leads to $n_1/L_1\sqrt{\rho_1} = n_2/L_2\sqrt{\rho_2}$, where $A$ has been canceled from both sides. The ratio of the integers is

$$\frac{n_2}{n_1} = \frac{L_2\sqrt{\rho_2}}{L_1\sqrt{\rho_1}} = \frac{(0.866\,\text{m})\sqrt{7.80\times10^3\ \text{kg/m}^3}}{(0.600\,\text{m})\sqrt{2.60\times10^3\ \text{kg/m}^3}} = 2.50.$$

The smallest integers that have this ratio are $n_1 = 2$ and $n_2 = 5$. The frequency is

$$f = n_1 v_1/2L_1 = (n_1/2L_1)\sqrt{\tau/\rho_1 A}.$$

The tension is provided by the hanging block and is $\tau = mg$, where $m$ is the mass of the block. Thus,

$$f = \frac{n_1}{2L_1}\sqrt{\frac{mg}{\rho_1 A}} = \frac{2}{2(0.600\,\text{m})}\sqrt{\frac{(10.0\,\text{kg})(9.80\,\text{m/s}^2)}{(2.60\times10^3\,\text{kg/m}^3)(1.00\times10^{-6}\,\text{m}^2)}} = 324\,\text{Hz}.$$

(b) The standing wave pattern has two loops in the aluminum section and five loops in the steel section, or seven loops in all. There are eight nodes, counting the end points.

60. With the string fixed on both ends, using Eq. 16-66 and Eq. 16-26, the resonant frequencies can be written as

$$f = \frac{nv}{2L} = \frac{n}{2L}\sqrt{\frac{\tau}{\mu}} = \frac{n}{2L}\sqrt{\frac{mg}{\mu}}, \qquad n = 1, 2, 3, \ldots$$

The mass that allows the oscillator to set up the nth harmonic on the string is

$$m = \frac{4L^2 f^2 \mu}{n^2 g}.$$

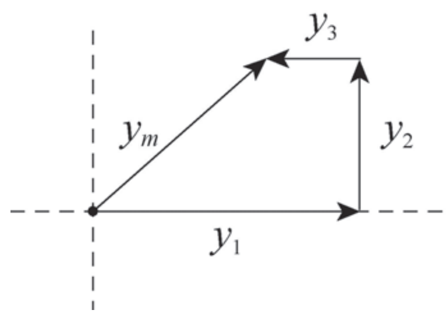
Thus, we see that the block mass is inversely proportional to the harmonic number squared. Thus, if the 447 gram block corresponds to harmonic number $n$ then

$$\frac{447}{286.1} = \frac{(n+1)^2}{n^2} = \frac{n^2 + 2n + 1}{n^2} = 1 + \frac{2n+1}{n^2}.$$

Therefore, $\frac{447}{286.1} - 1 = 0.5624$ must equal an odd integer ($2n + 1$) divided by a squared integer ($n^2$). That is, multiplying 0.5624 by a square (such as 1, 4, 9, 16, etc) should give us a number very close (within experimental uncertainty) to an odd number (1, 3, 5, …). Trying this out in succession (starting with multiplication by 1, then by 4, …), we find that multiplication by 16 gives a value very close to 9; we conclude $n = 4$ (so $n^2 = 16$ and $2n + 1 = 9$). Plugging $m = 0.447$ kg, $n = 4$, and the other values given in the problem, we find

$$\mu = 0.000845 \text{ kg/m} = 0.845 \text{ g/m}.$$

61. (a) The phasor diagram is shown here: $y_1$, $y_2$, and $y_3$ represent the original waves and $y_m$ represents the resultant wave.




The horizontal component of the resultant is $y_{mh} = y_1 - y_3 = y_1 - y_1/3 = 2y_1/3$. The vertical component is $y_{mv} = y_2 = y_1/2$. The amplitude of the resultant is

$$y_m = \sqrt{y_{mh}^2 + y_{mv}^2} = \sqrt{\left(\frac{2y_1}{3}\right)^2 + \left(\frac{y_1}{2}\right)^2} = \frac{5}{6}y_1 = 0.83y_1.$$

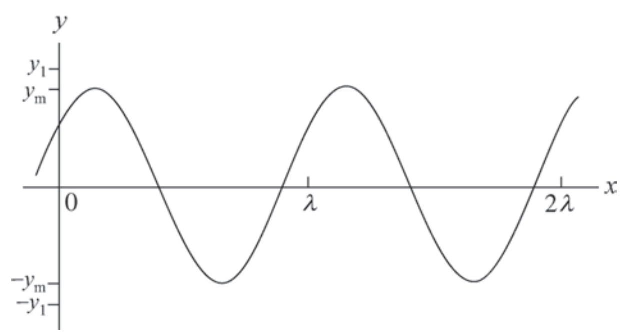
(b) The phase constant for the resultant is

$$\phi = \tan^{-1}\left(\frac{y_{mv}}{y_{mh}}\right) = \tan^{-1}\left(\frac{y_1/2}{2y_1/3}\right) = \tan^{-1}\left(\frac{3}{4}\right) = 0.644 \text{ rad} = 37°.$$

(c) The resultant wave is

$$y = \frac{5}{6}y_1 \sin(kx - \omega t + 0.644 \text{ rad}).$$

The graph below shows the wave at time $t = 0$. As time goes on it moves to the right with speed $v = \omega/k$.



62. Setting $x = 0$ in $y = y_m \sin(kx - \omega t + \phi)$ gives $y = y_m \sin(-\omega t + \phi)$ as the function being plotted in the graph. We note that it has a positive "slope" (referring to its $t$-derivative) at $t = 0$:

$$\frac{dy}{dt} = \frac{d\, y_m \sin(-\omega t + \phi)}{dt} = -y_m \omega \cos(-\omega t + \phi) > 0 \text{ at } t = 0.$$

This implies that $-\cos(\phi) > 0$ and consequently that $\phi$ is in either the second or third quadrant. The graph shows (at $t = 0$) $y = 2.00$ mm, and (at some later $t$) $y_m = 6.00$ mm. Therefore,

$$y = y_m \sin(-\omega t + \phi)\Big|_{t=0} \quad \Rightarrow \quad \phi = \sin^{-1}\left(\tfrac{1}{3}\right) = 0.34 \text{ rad} \quad \text{or} \quad 2.8 \text{ rad}$$

(bear in mind that $\sin(\theta) = \sin(\pi - \theta)$), and we must choose $\phi = 2.8$ rad because this is about 161° and is in second quadrant. Of course, this answer added to $2n\pi$ is still a valid answer (where n is any integer), so that, for example, $\phi = 2.8 - 2\pi = -3.48$ rad is also an acceptable result.

63. We compare the resultant wave given with the standard expression (Eq. 16–52) to obtain $k = 20\,\text{m}^{-1} = 2\pi/\lambda, 2y_m \cos\left(\tfrac{1}{2}\phi\right) = 3.0\,\text{mm}$, and $\tfrac{1}{2}\phi = 0.820\,\text{rad}$.

(a) Therefore, $\lambda = 2\pi/k = 0.31$ m.

(b) The phase difference is $\phi = 1.64$ rad.

(c) And the amplitude is $y_m = 2.2$ mm.

64. Setting $x = 0$ in $a_y = -\omega^2 y$ (see the solution to part (b) of Sample Problem 16-2) where $y = y_m \sin(kx - \omega t + \phi)$ gives $a_y = -\omega^2 y_m \sin(-\omega t + \phi)$ as the function being plotted in the graph. We note that it has a negative "slope" (referring to its $t$-derivative) at $t = 0$:

$$\frac{d a_y}{d t} = \frac{d(-\omega^2 y_m \sin(-\omega t + \phi))}{d t} = y_m \omega^3 \cos(-\omega t + \phi) < 0 \text{ at } t = 0.$$

This implies that $\cos\phi < 0$ and consequently that $\phi$ is in either the second or third quadrant. The graph shows (at t = 0) $a_y = -100$ m/s², and (at another $t$) $a_{max} = 400$ m/s². Therefore,

$$a_y = -a_{max} \sin(-\omega t + \phi)\Big|_{t=0} \quad \Longrightarrow \quad \phi = \sin^{-1}\left(\tfrac{1}{4}\right) = 0.25 \text{ rad} \quad \text{or} \quad 2.9 \text{ rad}$$

(bear in mind that $\sin\theta = \sin(\pi - \theta)$), and we must choose $\phi = 2.9$ rad because this is about 166° and is in the second quadrant. Of course, this answer added to $2n\pi$ is still a valid answer (where n is any integer), so that, for example, $\phi = 2.9 - 2\pi = -3.4$ rad is also an acceptable result.

65. We note that

$$dy/dt = -\omega\cos(kx - \omega t + \phi),$$

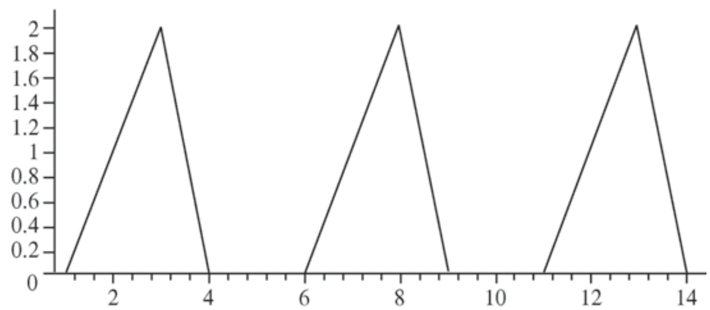
which we will refer to as $u(x,t)$. so that the ratio of the function $y(x,t)$ divided by $u(x,t)$ is $-\tan(kx - \omega t + \phi)/\omega$. With the given information (for $x = 0$ and $t = 0$) then we can take the inverse tangent of this ratio to solve for the phase constant:

$$\phi = \tan^{-1}\left(\frac{-\omega\, y(0,0)}{u(0,0)}\right) = \tan^{-1}\left(\frac{-(440)(0.0045)}{-0.75}\right) = 1.2 \text{ rad.}$$

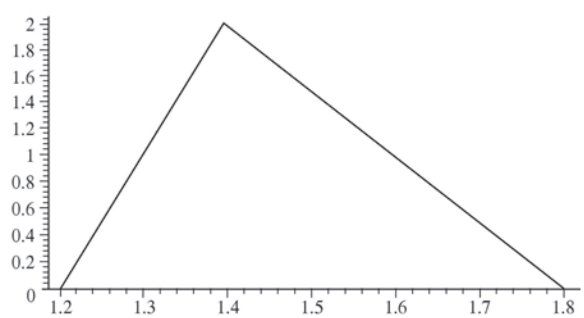
66. (a) Recalling the discussion in §16-5, we see that the speed of the wave given by a function with argument $x$ – 5.0$t$ (where $x$ is in centimeters and $t$ is in seconds) must be 5.0 cm/s.

(b) In part (c), we show several "snapshots" of the wave: the one on the left is as shown in Figure 16–48 (at $t$ = 0), the middle one is at $t$ = 1.0 s, and the rightmost one is at $t$ = 2.0 s. It is clear that the wave is traveling to the right (the +$x$ direction).

(c) The third picture in the sequence below shows the pulse at 2.0 s. The horizontal scale (and, presumably, the vertical one also) is in centimeters.




(d) The leading edge of the pulse reaches $x$ = 10 cm at $t$ = (10 – 4.0)/5 = 1.2 s. The particle (say, of the string that carries the pulse) at that location reaches a maximum displacement $h$ = 2 cm at $t$ = (10 – 3.0)/5 = 1.4 s. Finally, the trailing edge of the pulse departs from $x$ = 10 cm at $t$ = (10 – 1.0)/5 = 1.8 s. Thus, we find for $h(t)$ at $x$ = 10 cm (with the horizontal axis, $t$, in seconds):



67. (a) The displacement of the string is assumed to have the form $y(x, t) = y_m \sin(kx - \omega t)$. The velocity of a point on the string is

$$u(x, t) = \partial y/\partial t = -\omega y_m \cos(kx - \omega t)$$

and its maximum value is $u_m = \omega y_m$. For this wave the frequency is $f = 120$ Hz and the angular frequency is $\omega = 2\pi f = 2\pi$ (120 Hz) = 754 rad/s. Since the bar moves through a distance of 1.00 cm, the amplitude is half of that, or $y_m = 5.00 \times 10^{-3}$ m. The maximum speed is

$$u_m = (754 \text{ rad/s})\,(5.00 \times 10^{-3} \text{ m}) = 3.77 \text{ m/s}.$$

(b) Consider the string at coordinate $x$ and at time $t$ and suppose it makes the angle $\theta$ with the $x$ axis. The tension is along the string and makes the same angle with the $x$ axis. Its transverse component is $\tau_{\text{trans}} = \tau \sin\theta$. Now $\theta$ is given by $\tan\theta = \partial y/\partial x = k y_m \cos(kx - \omega t)$ and its maximum value is given by $\tan\theta_m = k y_m$. We must calculate the angular wave number $k$. It is given by $k = \omega/v$, where $v$ is the wave speed. The wave speed is given by $v = \sqrt{\tau/\mu}$, where $\tau$ is the tension in the rope and $\mu$ is the linear mass density of the rope. Using the data given,

$$v = \sqrt{\frac{90.0\,\text{N}}{0.120\,\text{kg/m}}} = 27.4\,\text{m/s}$$

and

$$k = \frac{754\,\text{rad/s}}{27.4\,\text{m/s}} = 27.5\,\text{m}^{-1}.$$

Thus,

$$\tan\theta_m = (27.5\,\text{m}^{-1})(5.00\times10^{-3}\,\text{m}) = 0.138$$

and $\theta = 7.83°$. The maximum value of the transverse component of the tension in the string is

$$\tau_{\text{trans}} = (90.0 \text{ N}) \sin 7.83° = 12.3 \text{ N}.$$

We note that $\sin\theta$ is nearly the same as $\tan\theta$ because $\theta$ is small. We can approximate the maximum value of the transverse component of the tension by $\tau k y_m$.

(c) We consider the string at $x$. The transverse component of the tension pulling on it due to the string to the left is $-\tau(\partial y/\partial x) = -\tau k y_m \cos(kx - \omega t)$ and it reaches its maximum value when $\cos(kx - \omega t) = -1$. The wave speed is

$$u = \partial y/\partial t = -\omega y_m \cos(kx - \omega t)$$

and it also reaches its maximum value when $\cos(kx - \omega t) = -1$. The two quantities reach their maximum values at the same value of the phase. When $\cos(kx - \omega t) = -1$ the value of $\sin(kx - \omega t)$ is zero and the displacement of the string is $y = 0$.

(d) When the string at any point moves through a small displacement $\Delta y$, the tension does work $\Delta W = \tau_{\text{trans}}\,\Delta y$. The rate at which it does work is

$$P = \frac{\Delta W}{\Delta t} = \tau_{\text{trans}}\frac{\Delta y}{\Delta t} = \tau_{\text{trans}}u.$$

$P$ has its maximum value when the transverse component $\tau_{\text{trans}}$ of the tension and the string speed $u$ have their maximum values. Hence the maximum power is (12.3 N)(3.77 m/s) = 46.4 W.

(e) As shown above $y = 0$ when the transverse component of the tension and the string speed have their maximum values.

(f) The power transferred is zero when the transverse component of the tension and the string speed are zero.

(g) $P = 0$ when $\cos(kx - \omega t) = 0$ and $\sin(kx - \omega t) = \pm 1$ at that time. The string displacement is $y = \pm y_m = \pm 0.50$ cm.

68. We use Eq. 16-52 in interpreting the figure.

(a) Since $y'$ = 6.0 mm when $\phi = 0$, then Eq. 16-52 can be used to determine $y_m$ = 3.0 mm.

(b) We note that $y'$ = 0 when the shift distance is 10 cm; this occurs because $\cos(\phi/2) = 0$ there $\Rightarrow$ $\phi = \pi$ rad or ½ cycle. Since a full cycle corresponds to a distance of one full wavelength, this ½ cycle shift corresponds to a distance of $\lambda/2$. Therefore, $\lambda$ = 20 cm $\Rightarrow$ $k = 2\pi/\lambda = 31\ \text{m}^{-1}$.

(c) Since $f$ = 120 Hz, $\omega = 2\pi f$ = 754 rad/s $\approx 7.5\times10^2$ rad/s.

(d) The sign in front of $\omega$ is minus since the waves are traveling in the $+x$ direction.

The results may be summarized as $y = (3.0\ \text{mm}) \sin[(31.4\ \text{m}^{-1})x - (754\ \text{s}^{-1})t]$] (this applies to each wave when they are in phase).

69. (a) We take the form of the displacement to be $y(x, t) = y_m \sin(kx - \omega t)$. The speed of a point on the cord is

$$u(x, t) = \partial y/\partial t = -\omega y_m \cos(kx - \omega t),$$

and its maximum value is $u_m = \omega y_m$. The wave speed, on the other hand, is given by $v = \lambda/T = \omega/k$. The ratio is

$$\frac{u_m}{v} = \frac{\omega y_m}{\omega / k} = k y_m = \frac{2\pi y_m}{\lambda}.$$

(b) The ratio of the speeds depends only on the ratio of the amplitude to the wavelength. Different waves on different cords have the same ratio of speeds if they have the same amplitude and wavelength, regardless of the wave speeds, linear densities of the cords, and the tensions in the cords.

70. We write the expression for the displacement in the form $y(x, t) = y_m \sin(kx - \omega t)$.

(a) The amplitude is $y_m$ = 2.0 cm = 0.020 m, as given in the problem.

(b) The angular wave number $k$ is $k = 2\pi/\lambda = 2\pi/(0.10 \text{ m}) = 63 \text{ m}^{-1}$

(c) The angular frequency is $\omega = 2\pi f = 2\pi(400 \text{ Hz}) = 2510 \text{ rad/s} = 2.5\times10^3 \text{ rad/s}$.

(d) A minus sign is used before the $\omega t$ term in the argument of the sine function because the wave is traveling in the positive $x$ direction.

Using the results above, the wave may be written as

$$y(x,t) = (2.00\,\text{cm})\sin\left(\left(62.8\,\text{m}^{-1}\right)x - \left(2510\,\text{s}^{-1}\right)t\right).$$

(e) The (transverse) speed of a point on the cord is given by taking the derivative of $y$:

$$u(x,t) = \frac{\partial y}{\partial t} = -\omega y_m \cos(kx - \omega t)$$

which leads to a maximum speed of $u_m = \omega y_m$ = (2510 rad/s)(0.020 m) = 50 m/s.

(f) The speed of the wave is

$$v = \frac{\lambda}{T} = \frac{\omega}{k} = \frac{2510\,\text{rad/s}}{62.8\,\text{rad/m}} = 40\,\text{m/s}.$$

71. (a) The amplitude is $y_m$ = 1.00 cm = 0.0100 m, as given in the problem.

(b) Since the frequency is $f$ = 550 Hz, the angular frequency is $\omega = 2\pi f = 3.46\times10^3$ rad/s.

(c) The angular wave number is $k = \omega / v = (3.46\times10^3 \text{ rad/s})/(330 \text{ m/s}) = 10.5 \text{ rad/m}$.

(d) Since the wave is traveling in the $-x$ direction, the sign in front of $\omega$ is plus and the argument of the trig function is $kx + \omega t$.

The results may be summarized as

$$
\begin{aligned}
y(x,t) &= y_\mathrm{m} \sin(kx+\omega t) = y_\mathrm{m} \sin\left[2\pi f\left(\frac{x}{v}+t\right)\right] \\
&= (0.010\,\mathrm{m})\sin\left[2\pi(550\,\mathrm{Hz})\left(\frac{x}{330\,\mathrm{m/s}}+t\right)\right] \\
&= (0.010\,\mathrm{m})\ \sin[(10.5\text{ rad/s})\ x + (3.46\times10^3\text{ rad/s})t].
\end{aligned}
$$

72. We orient one phasor along the *x* axis with length 3.0 mm and angle 0 and the other at 70° (in the first quadrant) with length 5.0 mm. Adding the components, we obtain

$$(3.0\ \text{mm}) + (5.0\ \text{mm})\cos(70°) = 4.71\text{mm}\ \ \text{along } x \text{ axis}$$
$$(5.0\ \text{mm})\sin(70°) = 4.70\ \text{mm}\ \ \text{along } y \text{ axis.}$$

(a) Thus, amplitude of the resultant wave is $\sqrt{(4.71\ \text{mm})^2 + (4.70\ \text{mm})^2} = 6.7\,\text{mm}$.

(b) And the angle (phase constant) is $\tan^{-1}$ (4.70/4.71) = 45°.

73. (a) Using $v = f\lambda$, we obtain

$$f = \frac{240\,\text{m/s}}{3.2\,\text{m}} = 75\ \text{Hz}.$$

(b) Since frequency is the reciprocal of the period, we find

$$T = \frac{1}{f} = \frac{1}{75\,\text{Hz}} = 0.0133\,\text{s} \approx 13\,\text{ms}.$$

74. By Eq. 16–66, the higher frequencies are integer multiples of the lowest (the fundamental).

(a) The frequency of the second harmonic is $f_2 = 2(440) = 880$ Hz.

(b) The frequency of the third harmonic is and $f_3 = 3(440) = 1320$ Hz.

75. We make use of Eq. 16–65 with $L$ = 120 cm.

(a) The longest wavelength for waves traveling on the string if standing waves are to be set up is $\lambda_1 = 2L/1 = 240$ cm.

(b) The second longest wavelength for waves traveling on the string if standing waves are to be set up is $\lambda_2 = 2L/2 = 120$ cm.

(c) The third longest wavelength for waves traveling on the string if standing waves are to be set up is $\lambda_3 = 2L/3 = 80.0$ cm.

The three standing waves are shown below:

76. (a) At $x$ = 2.3 m and $t$ = 0.16 s the displacement is

$$y(x,t) = 0.15\sin\left[(0.79)(2.3) - 13(0.16)\right]\text{m} = -0.039\,\text{m}.$$

(b) We choose $y_m$ = 0.15 m, so that there would be nodes (where the wave amplitude is zero) in the string as a result.

(c) The second wave must be traveling with the same speed and frequency. This implies $k = 0.79\ \text{m}^{-1}$,

(d) and $\omega = 13$ rad/s .

(e) The wave must be traveling in $-x$ direction, implying a plus sign in front of $\omega$.

Thus, its general form is $y'(x,t)$ = (0.15 m)sin(0.79$x$ + 13$t$).

(f) The displacement of the standing wave at $x$ = 2.3 m and $t$ = 0.16 s is

$$y(x,t) = -0.039\,\text{m} + (0.15\,\text{m})\sin[(0.79)(2.3) + 13(0.16)] = -0.14\,\text{m}.$$

77. (a) The wave speed is

$$v = \sqrt{\frac{\tau}{\mu}} = \sqrt{\frac{120\,\text{N}}{8.70\times10^{-3}\,\text{kg}/1.50\,\text{m}}} = 144\ \text{m/s}.$$

(b) For the one-loop standing wave we have $\lambda_1 = 2L = 2(1.50\text{ m}) = 3.00\text{ m}$.

(c) For the two-loop standing wave $\lambda_2 = L = 1.50\text{ m}$.

(d) The frequency for the one-loop wave is $f_1 = v/\lambda_1 = (144\text{ m/s})/(3.00\text{ m}) = 48.0\text{ Hz}$.

(e) The frequency for the two-loop wave is $f_2 = v/\lambda_2 = (144\text{ m/s})/(1.50\text{ m}) = 96.0\text{ Hz}$.

78. We use $P = \frac{1}{2}\mu v\omega^2 y_m^2 \propto vf^2 \propto \sqrt{\tau} f^2$.

(a) If the tension is quadrupled, then $P_2 = P_1\sqrt{\frac{\tau_2}{\tau_1}} = P_1\sqrt{\frac{4\tau_1}{\tau_1}} = 2P_1$.

(b) If the frequency is halved, then $P_2 = P_1\left(\frac{f_2}{f_1}\right)^2 = P_1\left(\frac{f_1/2}{f_1}\right)^2 = \frac{1}{4}P_1$.

79. We use Eq. 16-2, Eq. 16-5, Eq. 16-9, Eq. 16-13, and take the derivative to obtain the transverse speed $u$.

(a) The amplitude is $y_m$ = 2.0 mm.

(b) Since $\omega$ = 600 rad/s, the frequency is found to be $f$ = 600/2π ≈ 95 Hz.

(c) Since $k$ = 20 rad/m, the velocity of the wave is $v = \omega/k$ = 600/20 = 30 m/s in the $+x$ direction.

(d) The wavelength is λ = 2π/$k$ ≈ 0.31 m, or 31 cm.

(e) We obtain

$$u = \frac{dy}{dt} = -\omega y_m \cos(kx - \omega t) \Rightarrow u_m = \omega y_m$$

so that the maximum transverse speed is $u_m$ = (600)(2.0) = 1200 mm/s, or 1.2 m/s.

80. (a) Since the string has four loops its length must be two wavelengths. That is, $\lambda = L/2$, where $\lambda$ is the wavelength and $L$ is the length of the string. The wavelength is related to the frequency $f$ and wave speed $v$ by $\lambda = v/f$, so $L/2 = v/f$ and

$$L = 2v/f = 2(400 \text{ m/s})/(600 \text{ Hz}) = 1.3 \text{ m}.$$

(b) We write the expression for the string displacement in the form $y = y_m \sin(kx) \cos(\omega t)$, where $y_m$ is the maximum displacement, $k$ is the angular wave number, and $\omega$ is the angular frequency. The angular wave number is

$$k = 2\pi/\lambda = 2\pi f/v = 2\pi(600 \text{ Hz})/(400 \text{ m/s}) = 9.4\text{m}^{-1}$$

and the angular frequency is

$$\omega = 2\pi f = 2\pi(600 \text{ Hz}) = 3800 \text{ rad/s}.$$

With $y_m$ = 2.0 mm, the displacement is given by

$$y(x,t) = (2.0\,\text{mm})\sin[(9.4\,\text{m}^{-1})x]\cos[(3800\,\text{s}^{-1})t].$$

81. To oscillate in four loops means $n = 4$ in Eq. 16-65 (treating both ends of the string as effectively "fixed"). Thus, $\lambda = 2(0.90 \text{ m})/4 = 0.45$ m. Therefore, the speed of the wave is $v = f\lambda = 27$ m/s. The mass-per-unit-length is

$$\mu = m/L = (0.044 \text{ kg})/(0.90 \text{ m}) = 0.049 \text{ kg/m}.$$

Thus, using Eq. 16-26, we obtain the tension:

$$\tau = v^2 \mu = (27 \text{ m/s})^2(0.049 \text{ kg/m}) = 36 \text{ N}.$$

82. (a) This distance is determined by the longitudinal speed:

$$d_\ell = v_\ell t = (2000\,\text{m/s})\left(40\times10^{-6}\,\text{s}\right) = 8.0\times10^{-2}\,\text{m}.$$

(b) Assuming the acceleration is constant (justified by the near-straightness of the curve $a = 300/40 \times 10^{-6}$) we find the stopping distance $d$:

$$v^2 = v_o^2 + 2ad \Rightarrow d = \frac{(300)^2\left(40\times10^{-6}\right)}{2(300)}$$

which gives $d = 6.0\times10^{-3}$ m. This and the radius $r$ form the legs of a right triangle (where $r$ is opposite from $\theta = 60°$). Therefore,

$$\tan 60° = \frac{r}{d} \Rightarrow r = d\tan 60° = 1.0\times10^{-2}\,\text{m}.$$

83. (a) Let the cross-sectional area of the wire be $A$ and the density of steel be $\rho$. The tensile stress is given by $\tau/A$ where $\tau$ is the tension in the wire. Also, $\mu = \rho A$. Thus,

$$v_{\max} = \sqrt{\frac{\tau_{\max}}{\mu}} = \sqrt{\frac{\tau_{\max}/A}{\rho}} = \sqrt{\frac{7.00\times10^{8}\ \text{N/m}^2}{7800\,\text{kg/m}^3}} = 3.00\times10^{2}\ \text{m/s}$$

(b) The result does not depend on the diameter of the wire.

84. (a) Let the displacements of the wave at ($y$,$t$) be $z(y,t)$. Then

$$z(y,t) = z_m \sin(ky - \omega t),$$

where $z_m$ = 3.0 mm, $k$ = 60 cm$^{-1}$, and $\omega = 2\pi/T = 2\pi/0.20\text{ s} = 10\pi\text{ s}^{-1}$. Thus

$$z(y,t) = (3.0\,\text{mm})\sin\left[\left(60\,\text{cm}^{-1}\right)y - \left(10\pi\,\text{s}^{-1}\right)t\right].$$

(b) The maximum transverse speed is $u_m = \omega z_m = (2\pi/0.20\,\text{s})(3.0\,\text{mm}) = 94\,\text{mm/s}$.

85. (a) With length in centimeters and time in seconds, we have

$$u = \frac{dy}{dt} = -60\pi \cos\left(\frac{\pi x}{8} - 4\pi t\right).$$

Thus, when $x = 6$ and $t = \frac{1}{4}$, we obtain

$$u = -60\pi \cos\frac{-\pi}{4} = \frac{-60\pi}{\sqrt{2}} = -133$$

so that the *speed* there is 1.33 m/s.

(b) The numerical coefficient of the cosine in the expression for $u$ is $-60\pi$. Thus, the maximum *speed* is 1.88 m/s.

(c) Taking another derivative,

$$a = \frac{du}{dt} = -240\pi^2 \sin\left(\frac{\pi x}{8} - 4\pi t\right)$$

so that when $x = 6$ and $t = \frac{1}{4}$ we obtain $a = -240\pi^2 \sin(-\pi/4)$ which yields $a = 16.7\ \text{m/s}^2$.

(d) The numerical coefficient of the sine in the expression for $a$ is $-240\pi^2$. Thus, the maximum acceleration is $23.7\ \text{m/s}^2$.

86. Repeating the steps of Eq. 16-47 $\rightarrow$ Eq. 16-53, but applying

$$\cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

(see Appendix E) instead of Eq. 16-50, we obtain $y' = [0.10\cos\pi x]\cos 4\pi t$, with SI units understood.

(a) For non-negative $x$, the smallest value to produce $\cos\pi x = 0$ is $x = 1/2$, so the answer is $x = 0.50$ m.

(b) Taking the derivative,

$$u' = \frac{dy'}{dt} = [0.10\cos\pi x](-4\pi\sin 4\pi t)$$

We observe that the last factor is zero when $t = 0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \ldots$ Thus, the value of the first time the particle at $x$=0 has zero velocity is $t = 0$.

(c) Using the result obtained in (b), the second time where the velocity at $x$ =0 vanishes would be $t = 0.25$ s,

(d) and the third time is $t = 0.50$ s.

87. (a) From the frequency information, we find $\omega = 2\pi f = 10\pi$ rad/s. A point on the rope undergoing simple harmonic motion (discussed in Chapter 15) has maximum speed as it passes through its "middle" point, which is equal to $y_m\omega$. Thus,

$$5.0 \text{ m/s} = y_m\omega \quad \Rightarrow \quad y_m = 0.16 \text{ m} \quad .$$

(b) Because of the oscillation being in the *fundamental* mode (as illustrated in Fig. 16-23(a) in the textbook), we have $\lambda = 2L = 4.0$ m. Therefore, the speed of waves along the rope is $v = f\lambda = 20$ m/s. Then, with $\mu = m/L = 0.60$ kg/m, Eq. 16-26 leads to

$$v = \sqrt{\frac{\tau}{\mu}} \quad \Rightarrow \quad \tau = \mu v^2 = 240 \text{ N} \approx 2.4\times10^2 \text{ N} .$$

(c) We note that for the fundamental, $k = 2\pi/\lambda = \pi/L$, and we observe that the anti-node having zero displacement at $t = 0$ suggests the use of sine instead of cosine for the simple harmonic motion factor. Now, *if* the fundamental mode is the only one present (so the amplitude calculated in part (a) is indeed the amplitude of the fundamental wave pattern) then we have

$$y = (0.16 \text{ m}) \sin\left(\frac{\pi x}{2}\right) \sin(10\pi t) = (0.16 \text{ m})\sin[(1.57 \text{ m}^{-1})x]\sin[(31.4 \text{ rad/s})t]$$

88. (a) The frequency is $f = 1/T = 1/4$ Hz, so $v = f\lambda = 5.0$ cm/s.

(b) We refer to the graph to see that the maximum transverse speed (which we will refer to as $u_m$) is 5.0 cm/s. Recalling from Ch. 11 the simple harmonic motion relation $u_m = y_m\omega = y_m 2\pi f$, we have

$$5.0 = y_m\left(2\pi\frac{1}{4}\right) \quad \Rightarrow \quad y_m = 3.2 \text{ cm.}$$

(c) As already noted, $f = 0.25$ Hz.

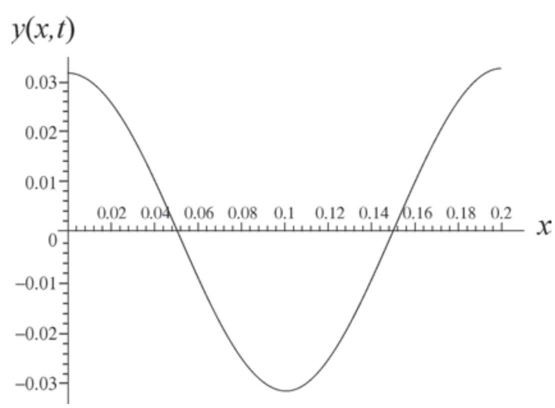
(d) Since $k = 2\pi/\lambda$, we have $k = 10\pi$ rad/m. There must be a sign difference between the $t$ and $x$ terms in the argument in order for the wave to travel to the right. The figure shows that at $x = 0$, the transverse velocity function is $0.050 \sin \pi t/2$. Therefore, the function $u(x,t)$ is

$$u(x,t) = 0.050 \sin\left(\frac{\pi}{2}t - 10\pi x\right)$$

with lengths in meters and time in seconds. Integrating this with respect to time yields

$$y(x,t) = -\frac{2(0.050)}{\pi}\cos\left(\frac{\pi}{2}t - 10\pi x\right) + C$$

where $C$ is an integration constant (which we will assume to be zero). The sketch of this function at $t = 2.0$ s for $0 \le x \le 0.20$ m is shown below.



89. (a) The wave speed is

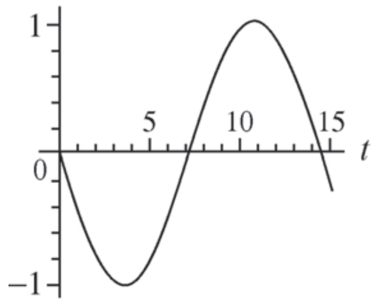
$$v=\sqrt{\frac{F}{\mu}}=\sqrt{\frac{k\Delta\ell}{m/(\ell+\Delta\ell)}}=\sqrt{\frac{k\Delta\ell(\ell+\Delta\ell)}{m}}.$$

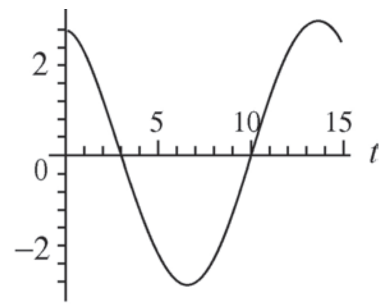
(b) The time required is

$$t=\frac{2\pi(\ell+\Delta\ell)}{v}=\frac{2\pi(\ell+\Delta\ell)}{\sqrt{k\Delta\ell(\ell+\Delta\ell)/m}}=2\pi\sqrt{\frac{m}{k}}\sqrt{1+\frac{\ell}{\Delta\ell}}.$$

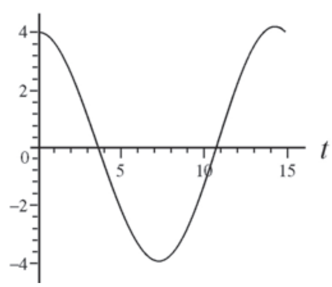
Thus if $\ell/\Delta\ell \gg 1$, then $t \propto \sqrt{\ell/\Delta\ell} \propto 1/\sqrt{\Delta\ell}$; and if $\ell/\Delta\ell \ll 1$, then $t \simeq 2\pi\sqrt{m/k} = \text{const.}$

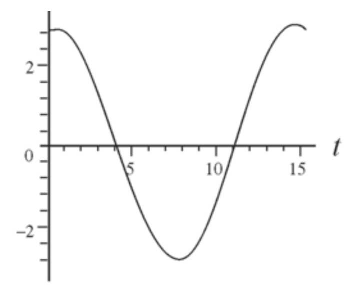
90. (a) The wave number for each wave is $k = 25.1$/m, which means $\lambda = 2\pi/k = 250.3$ mm. The angular frequency is $\omega = 440$/s; therefore, the period is $T = 2\pi/\omega = 14.3$ ms. We plot the superposition of the two waves $y = y_1 + y_2$ over the time interval $0 \le t \le 15$ ms. The first two graphs below show the oscillatory behavior at $x = 0$ (the graph on the left) and at $x = \lambda/8 \approx 31$ mm. The time unit is understood to be the millisecond and vertical axis ($y$) is in millimeters.

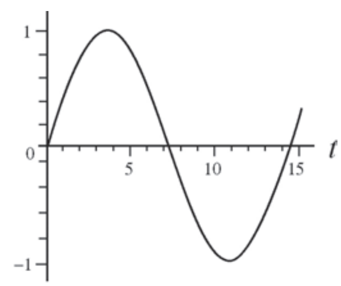



The following three graphs show the oscillation at $x = \lambda/4 = 62.6$ mm $\approx 63$ mm (graph on the left), at $x = 3\lambda/8 \approx 94$ mm (middle graph), and at $x = \lambda/2 \approx 125$ mm.




(b) We can think of wave $y_1$ as being made of two smaller waves going in the same direction, a wave $y_{1a}$ of amplitude 1.50 mm (the same as $y_2$) and a wave $y_{1b}$ of amplitude 1.00 mm. It is made clear in §16-12 that two equal-magnitude oppositely-moving waves form a standing wave pattern. Thus, waves $y_{1a}$ and $y_2$ form a standing wave, which leaves $y_{1b}$ as the remaining traveling wave. Since the argument of $y_{1b}$ involves the subtraction $kx - \omega t$, then $y_{1b}$ travels in the $+x$ direction.

(c) If $y_2$ (which travels in the $-x$ direction, which for simplicity will be called "leftward") had the larger amplitude, then the system would consist of a standing wave plus a leftward moving wave. A simple way to obtain such a situation would be to interchange the amplitudes of the given waves.

(d) Examining carefully the vertical axes, the graphs above certainly suggest that the largest amplitude of oscillation is $y_{max} = 4.0$ mm and occurs at $x = \lambda/4 = 62.6$ mm.

(e) The smallest amplitude of oscillation is $y_{min} = 1.0$ mm and occurs at $x = 0$ and at $x = \lambda/2 = 125$ mm.

(f) The largest amplitude can be related to the amplitudes of $y_1$ and $y_2$ in a simple way: $y_{max} = y_{1m} + y_{2m}$, where $y_{1m} = 2.5$ mm and $y_{2m} = 1.5$ mm are the amplitudes of the original traveling waves.

(g) The smallest amplitudes is $y_{min} = y_{1m} - y_{2m}$, where $y_{1m} = 2.5$ mm and $y_{2m} = 1.5$ mm are the amplitudes of the original traveling waves.

91. Using Eq. 16-50, we have

$$y' = \left[0.60\cos\frac{\pi}{6}\right]\sin\left(5\pi x - 200\pi t + \frac{\pi}{6}\right)$$

with length in meters and time in seconds (see Eq. 16-55 for comparison).

(a) The amplitude is seen to be

$$0.60\cos\frac{\pi}{6} = 0.3\sqrt{3} = 0.52\,\text{m}.$$

(b) Since $k = 5\pi$ and $\omega = 200\pi$, then (using Eq. 16-12) $v = \frac{\omega}{k} = 40\,\text{m/s}$.

(c) $k = 2\pi/\lambda$ leads to $\lambda = 0.40$ m.

92. (a) For visible light

$$f_{\min} = \frac{c}{\lambda_{\max}} = \frac{3.0\times10^{8}\ \text{m/s}}{700\times10^{-9}\,\text{m}} = 4.3\times10^{14}\ \text{Hz}$$

and

$$f_{\max} = \frac{c}{\lambda_{\min}} = \frac{3.0\times10^{8}\ \text{m/s}}{400\times10^{-9}\,\text{m}} = 7.5\times10^{14}\ \text{Hz}.$$

(b) For radio waves

$$\lambda_{\min} = \frac{c}{\lambda_{\max}} = \frac{3.0\times10^{8}\ \text{m/s}}{300\times10^{6}\,\text{Hz}} = 1.0\,\text{m}$$

and

$$\lambda_{\max} = \frac{c}{\lambda_{\min}} = \frac{3.0\times10^{8}\ \text{m/s}}{1.5\times10^{6}\,\text{Hz}} = 2.0\times10^{2}\,\text{m}.$$

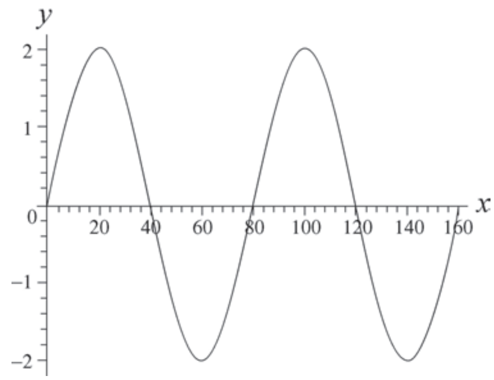
(c) For X rays

$$f_{\min} = \frac{c}{\lambda_{\max}} = \frac{3.0\times10^{8}\ \text{m/s}}{5.0\times10^{-9}\,\text{m}} = 6.0\times10^{16}\,\text{Hz}$$

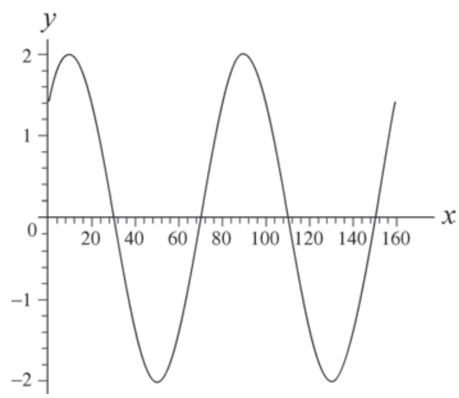
and

$$f_{\max} = \frac{c}{\lambda_{\min}} = \frac{3.0\times10^{8}\ \text{m/s}}{1.0\times10^{-11}\,\text{m}} = 3.0\times10^{19}\ \text{Hz}.$$

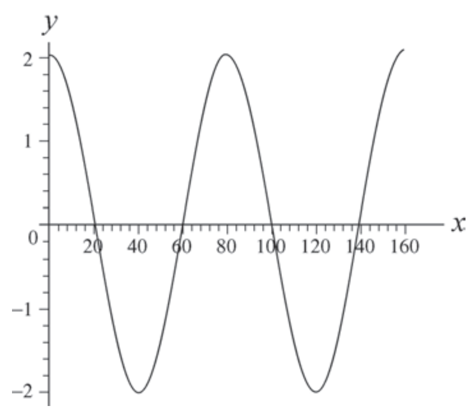
93. (a) Centimeters are to be understood as the length unit and seconds as the time unit. Making sure our (graphing) calculator is in radians mode, we find




(b) The previous graph is at $t$ = 0, and this next one is at $t$ = 0.050 s.




And the final one, shown below, is at $t$ = 0.010 s.




(c) The wave can be written as $y(x,t) = y_m \sin(kx + \omega t)$, where $v = \omega / k$ is the speed of propagation. From the problem statement, we see that $\omega = 2\pi / 0.40 = 5\pi$ rad/s and $k = 2\pi / 80 = \pi / 40$ rad/cm. This yields $v = 2.0 \times 10^2$ cm/s $= 2.0$ m/s

(d) These graphs (as well as the discussion in the textbook) make it clear that the wave is traveling in the –$x$ direction.

1. (a) When the speed is constant, we have $v = d/t$ where $v$ = 343 m/s is assumed. Therefore, with $t$ = 15/2 s being the time for sound to travel to the far wall we obtain $d$ = (343 m/s) × (15/2 s) which yields a distance of 2.6 km.

(b) Just as the $\frac{1}{2}$ factor in part (a) was 1/($n$ + 1) for $n$ = 1 reflection, so also can we write

$$d = (343\,\text{m/s})\left(\frac{15\,\text{s}}{n+1}\right) \quad \Rightarrow \quad n = \frac{(343)(15)}{d} - 1$$

for multiple reflections (with $d$ in meters). For $d$ = 25.7 m, we find $n$ = 199 $\approx 2.0\times10^2$ .

2. The time it takes for a soldier in the rear end of the column to switch from the left to the right foot to stride forward is $t$ = 1 min/120 = 1/120 min = 0.50 s. This is also the time for the sound of the music to reach from the musicians (who are in the front) to the rear end of the column. Thus the length of the column is

$$l = vt = (343\,\mathrm{m/s})(0.50\,\mathrm{s}) = 1.7 \times 10^2\,\mathrm{m}.$$

3. (a) The time for the sound to travel from the kicker to a spectator is given by $d/v$, where $d$ is the distance and $v$ is the speed of sound. The time for light to travel the same distance is given by $d/c$, where $c$ is the speed of light. The delay between seeing and hearing the kick is $\Delta t = (d/v) - (d/c)$. The speed of light is so much greater than the speed of sound that the delay can be approximated by $\Delta t = d/v$. This means $d = v\,\Delta t$. The distance from the kicker to spectator $A$ is

$$d_A = v\,\Delta t_A = (343\text{ m/s})(0.23\text{ s}) = 79\text{ m}.$$

(b) The distance from the kicker to spectator $B$ is $d_B = v\,\Delta t_B = (343\text{ m/s})(0.12\text{ s}) = 41\text{ m}$.

(c) Lines from the kicker to each spectator and from one spectator to the other form a right triangle with the line joining the spectators as the hypotenuse, so the distance between the spectators is

$$D = \sqrt{d_A^2 + d_B^2} = \sqrt{(79\text{ m})^2 + (41\text{ m})^2} = 89\text{ m}.$$

4. The density of oxygen gas is

$$\rho = \frac{0.0320\,\text{kg}}{0.0224\,\text{m}^3} = 1.43\,\text{kg/m}^3.$$

From $v = \sqrt{B/\rho}$ we find

$$B = v^2\rho = (317\,\text{m/s})^2\left(1.43\,\text{kg/m}^3\right) = 1.44\times10^5\ \text{Pa}.$$

5. Let $t_f$ be the time for the stone to fall to the water and $t_s$ be the time for the sound of the splash to travel from the water to the top of the well. Then, the total time elapsed from dropping the stone to hearing the splash is $t = t_f + t_s$. If $d$ is the depth of the well, then the kinematics of free fall gives

$$d = \frac{1}{2} g t_f^2 \quad \Rightarrow \quad t_f = \sqrt{2d/g}.$$

The sound travels at a constant speed $v_s$, so $d = v_s t_s$, or $t_s = d/v_s$. Thus the total time is $t = \sqrt{2d/g} + d/v_s$. This equation is to be solved for $d$. Rewrite it as $\sqrt{2d/g} = t - d/v_s$ and square both sides to obtain

$$2d/g = t^2 - 2(t/v_s)d + (1 + v_s^2)d^2.$$

Now multiply by $g v_s^2$ and rearrange to get

$$gd^2 - 2v_s(gt + v_s)d + g v_s^2 t^2 = 0.$$

This is a quadratic equation for $d$. Its solutions are

$$d = \frac{2v_s(gt + v_s) \pm \sqrt{4v_s^2(gt + v_s)^2 - 4g^2 v_s^2 t^2}}{2g}.$$

The physical solution must yield $d = 0$ for $t = 0$, so we take the solution with the negative sign in front of the square root. Once values are substituted the result $d = 40.7$ m is obtained.

6. Using Eqs. 16-13 and 17-3, the speed of sound can be expressed as

$$v = \lambda f = \sqrt{\frac{B}{\rho}},$$

where $B = -(dp/dV)/V$. Since $V, \lambda$ and $\rho$ are not changed appreciably, the frequency ratio becomes

$$\frac{f_s}{f_i} = \frac{v_s}{v_i} = \sqrt{\frac{B_s}{B_i}} = \sqrt{\frac{(dp/dV)_s}{(dp/dV)_i}}.$$

Thus, we have

$$\frac{(dV/dp)_s}{(dV/dp)_i} = \frac{B_i}{B_s} = \left(\frac{f_i}{f_s}\right)^2 = \left(\frac{1}{0.333}\right)^2 = 9.00.$$

7. If $d$ is the distance from the location of the earthquake to the seismograph and $v_s$ is the speed of the S waves then the time for these waves to reach the seismograph is $t_s$. = $d/v_s$. Similarly, the time for P waves to reach the seismograph is $t_p = d/v_p$. The time delay is

$$\Delta t = (d/v_s) - (d/v_p) = d(v_p - v_s)/v_s v_p,$$

so

$$d = \frac{v_s v_p \Delta t}{(v_p - v_s)} = \frac{(4.5\ \text{km/s})(8.0\,\text{km/s})(3.0\,\text{min})(60\,\text{s/min})}{8.0\,\text{km/s} - 4.5\,\text{km/s}} = 1.9\times10^3\ \text{km}.$$

We note that values for the speeds were substituted as given, in km/s, but that the value for the time delay was converted from minutes to seconds.

8. Let $\ell$ be the length of the rod. Then the time of travel for sound in air (speed $v_s$) will be $t_s = \ell / v_s$. And the time of travel for compressional waves in the rod (speed $v_r$) will be $t_r = \ell / v_r$. In these terms, the problem tells us that

$$t_s - t_r = 0.12\,\text{s} = \ell\left(\frac{1}{v_s} - \frac{1}{v_r}\right).$$

Thus, with $v_s$ = 343 m/s and $v_r = 15v_s$ = 5145 m/s, we find $\ell = 44\,\text{m}$.

9. (a) Using $\lambda = v/f$, where $v$ is the speed of sound in air and $f$ is the frequency, we find

$$\lambda = \frac{343\,\text{m/s}}{4.50\times10^{6}\ \text{Hz}} = 7.62\times10^{-5}\ \text{m}.$$

(b) Now, $\lambda = v/f$, where $v$ is the speed of sound in tissue. The frequency is the same for air and tissue. Thus

$$\lambda = (1500\ \text{m/s})/(4.50 \times 10^{6}\ \text{Hz}) = 3.33 \times 10^{-4}\ \text{m}.$$

10. (a) The amplitude of a sinusoidal wave is the numerical coefficient of the sine (or cosine) function: $p_m$ = 1.50 Pa.

(b) We identify $k = 0.9\pi$ and $\omega = 315\pi$ (in SI units), which leads to $f = \omega/2\pi$ = 158 Hz.

(c) We also obtain $\lambda = 2\pi/k$ = 2.22 m.

(d) The speed of the wave is $v = \omega/k$ = 350 m/s.

11. Without loss of generality we take $x = 0$, and let $t = 0$ be when $s = 0$. This means the phase is $\phi = -\pi/2$ and the function is $s = (6.0 \text{ nm})\sin(\omega t)$ at $x = 0$. Noting that $\omega = 3000$ rad/s, we note that at $t = \sin^{-1}(1/3)/\omega = 0.1133$ ms the displacement is $s = +2.0$ nm. Doubling that time (so that we consider the excursion from –2.0 nm to +2.0 nm) we conclude that the time required is 2(0.1133 ms) = 0.23 ms.

12. The key idea here is that the time delay $\Delta t$ is due to the distance $d$ that each wavefront must travel to reach your left ear ($L$) after it reaches your right ear ($R$).

(a) From the figure, we find $\Delta t = \frac{d}{v} = \frac{D\sin\theta}{v}$.

(b) Since the speed of sound in water is now $v_w$, with $\theta = 90°$, we have

$$\Delta t_w = \frac{D\sin 90°}{v_w} = \frac{D}{v_w}.$$

(c) The apparent angle can be found by substituting $D/v_w$ for $\Delta t$:

$$\Delta t = \frac{D\sin\theta}{v} = \frac{D}{v_w}.$$

Solving for $\theta$ with $v_w = 1482 \text{ m/s}$ (see Table 17-1), we obtain

$$\theta = \sin^{-1}\left(\frac{v}{v_w}\right) = \sin^{-1}\left(\frac{343 \text{ m/s}}{1482 \text{ m/s}}\right) = \sin^{-1}(0.231) = 13°$$

13. (a) Consider a string of pulses returning to the stage. A pulse which came back just before the previous one has traveled an extra distance of $2w$, taking an extra amount of time $\Delta t = 2w/v$. The frequency of the pulse is therefore

$$f = \frac{1}{\Delta t} = \frac{v}{2w} = \frac{343\,\text{m/s}}{2(0.75\,\text{m})} = 2.3 \times 10^{2}\ \text{Hz}.$$

(b) Since $f \propto 1/w$, the frequency would be higher if $w$ were smaller.

14. (a) The period is $T$ = 2.0 ms (or 0.0020 s) and the amplitude is $\Delta p_m$ = 8.0 mPa (which is equivalent to 0.0080 N/m$^2$). From Eq. 17-15 we get

$$s_m = \frac{\Delta p_m}{v\rho\omega} = \frac{\Delta p_m}{v\rho(2\pi/T)} = 6.1 \times 10^{-9}\ \text{m}.$$

where $\rho$ = 1.21 kg/m$^3$ and $v$ = 343 m/s.

(b) The angular wave number is $k = \omega/v = 2\pi/vT$ = 9.2 rad/m.

(c) The angular frequency is $\omega = 2\pi/T$ = 3142 rad/s $\approx 3.1 \times 10^3$ rad/s.

The results may be summarized as $s(x, t)$ = (6.1 nm) cos[(9.2 m$^{-1}$)$x$ – (3.1 × 10$^3$ s$^{-1}$)$t$].

(d) Using similar reasoning, but with the new values for density ($\rho'$ = 1.35 kg/m$^3$) and speed ($v'$ = 320 m/s), we obtain

$$s_m = \frac{\Delta p_m}{v'\rho'\omega} = \frac{\Delta p_m}{v'\rho'(2\pi/T)} = 5.9 \times 10^{-9}\ \text{m}.$$

(e) The angular wave number is $k = \omega/v' = 2\pi/v'T$ = 9.8 rad/m.

(f) The angular frequency is $\omega = 2\pi/T$ = 3142 rad/s $\approx 3.1 \times 10^3$ rad/s.

The new displacement function is $s(x, t)$ = (5.9 nm) cos[(9.8 m$^{-1}$)$x$ – (3.1 × 10$^3$ s$^{-1}$)$t$].

15. The problem says "At one instant.." and we choose that instant (without loss of generality) to be $t = 0$. Thus, the displacement of "air molecule $A$" at that instant is

$$s_A = +s_m = s_m\cos(kx_A - \omega t + \phi)\big|_{t=0} = s_m\cos(kx_A + \phi),$$

where $x_A = 2.00$ m. Regarding "air molecule $B$" we have

$$s_B = +\tfrac{1}{3}s_m = s_m\cos(kx_B - \omega t + \phi)\big|_{t=0} = s_m\cos(kx_B + \phi).$$

These statements lead to the following conditions:

$$kx_A + \phi = 0$$
$$kx_B + \phi = \cos^{-1}(1/3) = 1.231$$

where $x_B = 2.07$ m. Subtracting these equations leads to

$$k(x_B - x_A) = 1.231 \quad \Rightarrow \quad k = 17.6 \text{ rad/m}.$$

Using the fact that $k = 2\pi/\lambda$ we find $\lambda = 0.357$ m, which means

$$f = v/\lambda = 343/0.357 = 960 \text{ Hz}.$$

Another way to complete this problem (once $k$ is found) is to use $kv = \omega$ and then the fact that $\omega = 2\pi f$.

16. Let the separation between the point and the two sources (labeled 1 and 2) be $x_1$ and $x_2$, respectively. Then the phase difference is

$$\Delta\phi = \phi_1 - \phi_2 = 2\pi\left(\frac{x_1}{\lambda} + ft\right) - 2\pi\left(\frac{x_2}{\lambda} + ft\right) = \frac{2\pi(x_1 - x_2)}{\lambda} = \frac{2\pi(4.40\,\text{m} - 4.00\,\text{m})}{(330\,\text{m/s})/540\,\text{Hz}} = 4.12\,\text{rad}.$$

17. (a) The problem is asking at how many angles will there be "loud" resultant waves, and at how many will there be "quiet" ones? We note that at all points (at large distance from the origin) along the *x* axis there will be quiet ones; one way to see this is to note that the path-length difference (for the waves traveling from their respective sources) divided by wavelength gives the (dimensionless) value 3.5, implying a half-wavelength (180º) phase difference (destructive interference) between the waves. To distinguish the destructive interference along the +*x* axis from the destructive interference along the –*x* axis, we label one with +3.5 and the other –3.5. This labeling is useful in that it suggests that the complete enumeration of the quiet directions in the upper-half plane (including the *x* axis) is: –3.5, –2.5, –1.5, –0.5, +0.5, +1.5, +2.5, +3.5. Similarly, the complete enumeration of the loud directions in the upper-half plane is: –3, –2, –1, 0, +1, +2, +3. Counting also the "other" –3, –2, –1, 0, +1, +2, +3 values for the *lower*-half plane, then we conclude there are a total of 7 + 7 = 14 "loud" directions.

(b) The discussion about the "quiet" directions was started in part (a). The number of values in the list: –3.5, –2.5, –1.5, –0.5, +0.5, +1.5, +2.5, +3.5 along with –2.5, –1.5, –0.5, +0.5, +1.5, +2.5 (for the lower-half plane) is 14. There are 14 "quiet" directions.

18. At the location of the detector, the phase difference between the wave which traveled straight down the tube and the other one which took the semi-circular detour is

$$\Delta\phi = k\Delta d = \frac{2\pi}{\lambda}(\pi r - 2r).$$

For $r = r_{\min}$ we have $\Delta\phi = \pi$, which is the smallest phase difference for a destructive interference to occur. Thus,

$$r_{\min} = \frac{\lambda}{2(\pi - 2)} = \frac{40.0\,\text{cm}}{2(\pi - 2)} = 17.5\,\text{cm}.$$

19. Let $L_1$ be the distance from the closer speaker to the listener. The distance from the other speaker to the listener is $L_2 = \sqrt{L_1^2 + d^2}$ , where $d$ is the distance between the speakers. The phase difference at the listener is $\phi = 2\pi(L_2 - L_1)/\lambda$, where $\lambda$ is the wavelength.

For a minimum in intensity at the listener, $\phi = (2n + 1)\pi$, where $n$ is an integer. Thus,

$$\lambda = 2(L_2 - L_1)/(2n + 1).$$

The frequency is

$$f = \frac{v}{\lambda} = \frac{(2n+1)v}{2\left(\sqrt{L_1^2 + d^2} - L_1\right)} = \frac{(2n+1)(343\,\text{m/s})}{2\left(\sqrt{(3.75\,\text{m})^2 + (2.00\,\text{m})^2} - 3.75\,\text{m}\right)} = (2n+1)(343\,\text{Hz}).$$

Now 20,000/343 = 58.3, so $2n + 1$ must range from 0 to 57 for the frequency to be in the audible range. This means $n$ ranges from 0 to 28.

(a) The lowest frequency that gives minimum signal is ($n = 0$) $f_{\text{min},1} = 343$ Hz.

(b) The second lowest frequency is ($n = 1$) $f_{\text{min},2} = [2(1)+1]343 \text{ Hz} = 1029 \text{ Hz} = 3f_{\text{min},1}$. Thus, the factor is 3.

(c) The third lowest frequency is ($n$=2) $f_{\text{min},3} = [2(2)+1]343 \text{ Hz} = 1715 \text{ Hz} = 5f_{\text{min},1}$. Thus, the factor is 5.

For a maximum in intensity at the listener, $\phi = 2n\pi$, where $n$ is any positive integer. Thus $\lambda = (1/n)\left(\sqrt{L_1^2 + d^2} - L_1\right)$ and

$$f = \frac{v}{\lambda} = \frac{nv}{\sqrt{L_1^2 + d^2} - L_1} = \frac{n(343\,\text{m/s})}{\sqrt{(3.75\,\text{m})^2 + (2.00\,\text{m})^2} - 3.75\,\text{m}} = n(686\,\text{Hz}).$$

Since 20,000/686 = 29.2, $n$ must be in the range from 1 to 29 for the frequency to be audible.

(d) The lowest frequency that gives maximum signal is ($n$ =1) $f_{\text{max},1} = 686$ Hz.

(e) The second lowest frequency is ($n$ = 2) $f_{\text{max},2} = 2(686 \text{ Hz}) = 1372 \text{ Hz} = 2f_{\text{max},1}$. Thus, the factor is 2.

(f) The third lowest frequency is ($n$ = 3) $f_{\text{max},3} = 3(686 \text{ Hz}) = 2058 \text{ Hz} = 3f_{\text{max},1}$. Thus, the factor is 3.

20. (a) To be out of phase (and thus result in destructive interference if they superpose) means their path difference must be $\lambda/2$ (or $3\lambda/2$ or $5\lambda/2$ or …). Here we see their path difference is $L$, so we must have (in the least possibility) $L = \lambda/2$, or $q = L/\lambda = 0.5$.

(b) As noted above, the next possibility is $L = 3\lambda/2$, or $q = L/\lambda = 1.5$.

21. Building on the theory developed in §17 – 5, we set $\Delta L/\lambda = n - 1/2,\ n = 1, 2, \ldots$ in order to have destructive interference. Since $v = f\lambda$, we can write this in terms of frequency:

$$f_{\min,n} = \frac{(2n-1)v}{2\Delta L} = (n - 1/2)(286\ \text{Hz})$$

where we have used $v$ = 343 m/s (note the remarks made in the textbook at the beginning of the exercises and problems section) and $\Delta L$ = (19.5 – 18.3 ) m = 1.2 m.

(a) The lowest frequency that gives destructive interference is ($n$ = 1)

$$f_{\min,1} = (1 - 1/2)(286\ \text{Hz}) = 143\ \text{Hz}.$$

(b) The second lowest frequency that gives destructive interference is ($n$ = 2)

$$f_{\min,2} = (2 - 1/2)(286\ \text{Hz}) = 429\ \text{Hz} = 3(143\ \text{Hz}) = 3 f_{\min,1}.$$

So the factor is 3.

(c) The third lowest frequency that gives destructive interference is ($n$ = 3)

$$f_{\min,3} = (3 - 1/2)(286\ \text{Hz}) = 715\ \text{Hz} = 5(143\ \text{Hz}) = 5 f_{\min,1}.$$

So the factor is 5.

Now we set $\Delta L/\lambda = \frac{1}{2}$ (even numbers) — which can be written more simply as "(all integers $n$ = 1, 2,…)" — in order to establish constructive interference. Thus,

$$f_{\max,n} = \frac{nv}{\Delta L} = n(286\ \text{Hz}).$$

(d) The lowest frequency that gives constructive interference is ($n$ =1) $f_{\max,1} = (286\ \text{Hz})$.

(e) The second lowest frequency that gives constructive interference is ($n$ = 2)

$$f_{\max,2} = 2(286\ \text{Hz}) = 572\ \text{Hz} = 2 f_{\max,1}.$$

Thus, the factor is 2.

(f) The third lowest frequency that gives constructive interference is ($n$ = 3)

$$f_{\max,3} = 3(286\ \text{Hz}) = 858\ \text{Hz} = 3 f_{\max,1}.$$

Thus, the factor is 3.

22. (a) The problem indicates that we should ignore the decrease in sound amplitude which means that all waves passing through point *P* have equal amplitude. Their superposition at *P* if $d = \lambda/4$ results in a net effect of zero there since there are four sources (so the first and third are $\lambda/2$ apart and thus interfere destructively; similarly for the second and fourth sources).

(b) Their superposition at *P* if $d = \lambda/2$ also results in a net effect of zero there since there are an even number of sources (so the first and second being $\lambda/2$ apart will interfere destructively; similarly for the waves from the third and fourth sources).

(c) If $d = \lambda$ then the waves from the first and second sources will arrive at *P* in phase; similar observations apply to the second and third, and to the third and fourth sources. Thus, four waves interfere constructively there with net amplitude equal to $4s_m$.

23. (a) If point *P* is infinitely far away, then the small distance *d* between the two sources is of no consequence (they seem effectively to be the same distance away from *P*). Thus, there is no perceived phase difference.

(b) Since the sources oscillate in phase, then the situation described in part (a) produces fully constructive interference.

(c) For finite values of *x*, the difference in source positions becomes significant. The path lengths for waves to travel from $S_1$ and $S_2$ become now different. We interpret the question as asking for the behavior of the absolute value of the phase difference $|\Delta\phi|$, in which case any change from zero (the answer for part (a)) is certainly an increase.

The path length difference for waves traveling from $S_1$ and $S_2$ is

$$\Delta\ell = \sqrt{d^2 + x^2} - x \qquad \text{for} \quad x > 0.$$

The phase difference in "cycles" (in absolute value) is therefore

$$|\Delta\phi| = \frac{\Delta\ell}{\lambda} = \frac{\sqrt{d^2 + x^2} - x}{\lambda}.$$

Thus, in terms of λ, the phase difference is identical to the path length difference: $|\Delta\phi| = \Delta\ell > 0$. Consider $\Delta\ell = \lambda/2$. Then $\sqrt{d^2 + x^2} = x + \lambda/2$. Squaring both sides, rearranging, and solving, we find

$$x = \frac{d^2}{\lambda} - \frac{\lambda}{4}.$$

In general, if $\Delta\ell = \xi\lambda$ for some multiplier $\xi > 0$, we find

$$x = \frac{d^2}{2\xi\lambda} - \frac{1}{2}\xi\lambda = \frac{64.0}{\xi} - \xi$$

where we have used *d* = 16.0 m and λ = 2.00 m.

(d) For $\Delta\ell = 0.50\lambda$, or $\xi = 0.50$, we have $x = (64.0/0.50 - 0.50)\text{ m} = 127.5\text{ m} \approx 128\text{ m}$.

(e) For $\Delta\ell = 1.00\lambda$, or $\xi = 1.00$, we have $x = (64.0/1.00 - 1.00)\text{ m} = 63.0\text{ m}$.

(f) For $\Delta\ell = 1.50\lambda$, or $\xi = 1.50$, we have $x = (64.0/1.50 - 1.50)\text{ m} = 41.2\text{ m}$.

Note that since whole cycle phase differences are equivalent (as far as the wave superposition goes) to zero phase difference, then the $\xi$ = 1, 2 cases give constructive interference. A shift of a half-cycle brings "troughs" of one wave in superposition with "crests" of the other, thereby canceling the waves; therefore, the $\xi = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ cases produce destructive interference.

24. (a) Since intensity is power divided by area, and for an isotropic source the area may be written $A = 4\pi r^2$ (the area of a sphere), then we have

$$I = \frac{P}{A} = \frac{1.0\,\text{W}}{4\pi(1.0\,\text{m})^2} = 0.080\,\text{W/m}^2.$$

(b) This calculation may be done exactly as shown in part (a) (but with $r = 2.5$ m instead of $r = 1.0$ m), or it may be done by setting up a ratio. We illustrate the latter approach. Thus,

$$\frac{I'}{I} = \frac{P/4\pi(r')^2}{P/4\pi r^2} = \left(\frac{r}{r'}\right)^2$$

leads to $I' = (0.080\ \text{W/m}^2)(1.0/2.5)^2 = 0.013\ \text{W/m}^2$.

25. The intensity is the rate of energy flow per unit area perpendicular to the flow. The rate at which energy flow across every sphere centered at the source is the same, regardless of the sphere radius, and is the same as the power output of the source. If $P$ is the power output and $I$ is the intensity a distance $r$ from the source, then $P = IA = 4\pi r^2 I$, where $A$ $(= 4\pi r^2)$ is the surface area of a sphere of radius $r$. Thus

$$P = 4\pi(2.50 \text{ m})^2 (1.91 \times 10^{-4} \text{ W/m}^2) = 1.50 \times 10^{-2} \text{ W}.$$

26. Sample Problem 17-5 shows that a decibel difference $\Delta\beta$ is directly related to an intensity ratio (which we write as $\mathcal{R} = I'/I$ ). Thus,

$$\Delta\beta = 10\log(\mathcal{R}) \;\Rightarrow\; \mathcal{R} = 10^{\Delta\beta/10} = 10^{0.1} = 1.26.$$

27. The intensity is given by $I = \frac{1}{2}\rho v\omega^2 s_m^2$, where $\rho$ is the density of air, $v$ is the speed of sound in air, $\omega$ is the angular frequency, and $s_m$ is the displacement amplitude for the sound wave. Replace $\omega$ with $2\pi f$ and solve for $s_m$:

$$s_m = \sqrt{\frac{I}{2\pi^2 \rho v f^2}} = \sqrt{\frac{1.00\times10^{-6}\ \text{W/m}^2}{2\pi^2(1.21\,\text{kg/m}^3)(343\,\text{m/s})(300\,\text{Hz})^2}} = 3.68\times10^{-8}\ \text{m}.$$

28. (a) The intensity is given by $I = P/4\pi r^2$ when the source is "point-like." Therefore, at $r$ = 3.00 m,

$$I = \frac{1.00\times10^{-6}\ \text{W}}{4\pi(3.00\,\text{m})^2} = 8.84\times10^{-9}\ \text{W/m}^2.$$

(b) The sound level there is

$$\beta = 10\log\left(\frac{8.84\times10^{-9}\ \text{W/m}^2}{1.00\times10^{-12}\ \text{W/m}^2}\right) = 39.5\,\text{dB}.$$

29. (a) Let $I_1$ be the original intensity and $I_2$ be the final intensity. The original sound level is $\beta_1$ = (10 dB) log($I_1/I_0$) and the final sound level is $\beta_2$ = (10 dB) log($I_2/I_0$), where $I_0$ is the reference intensity. Since $\beta_2 = \beta_1$ + 30 dB which yields

$$(10 \text{ dB}) \log(I_2/I_0) = (10 \text{ dB}) \log(I_1/I_0) + 30 \text{ dB},$$

or

$$(10 \text{ dB}) \log(I_2/I_0) - (10 \text{ dB}) \log(I_1/I_0) = 30 \text{ dB}.$$

Divide by 10 dB and use log($I_2/I_0$) – log($I_1/I_0$) = log($I_2/I_1$) to obtain log($I_2/I_1$) = 3. Now use each side as an exponent of 10 and recognize that $10^{\log(I_2/I_1)} = I_2/I_1$. The result is $I_2/I_1 = 10^3$. The intensity is increased by a factor of $1.0\times10^3$.

(b) The pressure amplitude is proportional to the square root of the intensity so it is increased by a factor of $\sqrt{1000} \approx 32$.

30. (a) Eq. 17-29 gives the relation between sound level β and intensity *I*, namely

$$I = I_0 10^{(\beta/10\text{dB})} = (10^{-12}\,\text{W/m}^2)10^{(\beta/10\text{dB})} = 10^{-12+(\beta/10\text{dB})}\,\text{W/m}^2$$

Thus we find that for a $\beta$ = 70 dB level we have a high intensity value of $I_{\text{high}}$ = 10 $\mu$W/m$^2$.

(b) Similarly, for $\beta$ = 50 dB level we have a low intensity value of $I_{\text{low}}$ = 0.10 $\mu$W/m$^2$.

(c) Eq. 17-27 gives the relation between the displacement amplitude and *I*. Using the values for density and wave speed, we find $s_m$ = 70 nm for the high intensity case.

(d) Similarly, for the low intensity case we have $s_m$ = 7.0 nm.

We note that although the intensities differed by a factor of 100, the amplitudes differed by only a factor of 10.

31. We use $\beta = 10 \log(I/I_o)$ with $I_o = 1 \times 10^{-12}$ W/m$^2$ and Eq. 17–27 with $\omega = 2\pi f = 2\pi(260 \text{ Hz})$, $v = 343$ m/s and $\rho = 1.21$ kg/m$^3$.

$$I = I_o\left(10^{8.5}\right) = \frac{1}{2}\rho v\left(2\pi f\right)^2 s_m^2 \quad \Rightarrow \quad s_m = 7.6\times10^{-7} \text{ m} = 0.76\ \mu\text{m}.$$

32. (a) Since $\omega = 2\pi f$, Eq. 17-15 leads to

$$\Delta p_m = v\rho(2\pi f)s_m \quad \Rightarrow \quad s_m = \frac{1.13\times10^{-3}\text{ Pa}}{2\pi(1665\text{ Hz})(343\text{ m/s})(1.21\text{ kg/m}^3)}$$

which yields $s_m$ = 0.26 nm. The nano prefix represents $10^{-9}$. We use the speed of sound and air density values given at the beginning of the exercises and problems section in the textbook.

(b) We can plug into Eq. 17–27 or into its equivalent form, rewritten in terms of the pressure amplitude:

$$I = \frac{1}{2}\frac{(\Delta p_m)^2}{\rho v} = \frac{1}{2}\frac{(1.13\times10^{-3}\text{ Pa})^2}{(1.21\text{ kg/m}^3)(343\text{ m/s})} = 1.5\ \text{nW/m}^2.$$

33. We use $\beta = 10 \log (I/I_o)$ with $I_o = 1 \times 10^{-12}$ W/m$^2$ and $I = P/4\pi r^2$ (an assumption we are asked to make in the problem). We estimate $r \approx 0.3$ m (distance from knuckle to ear) and find

$$P \approx 4\pi(0.3\,\text{m})^2\left(1\times10^{-12}\ \text{W/m}^2\right)10^{6.2} = 2\times10^{-6}\ \text{W} = 2\ \mu\text{W}.$$

34. The difference in sound level is given by Eq. 17-37:

$$\Delta\beta = \beta_f - \beta_i = (10 \text{ db}) \log\left(\frac{I_f}{I_i}\right).$$

Thus, if $\Delta\beta = 5.0 \text{ db}$, then $\log(I_f / I_i) = 1/2$, which implies that $I_f = \sqrt{10} I_i$. On the other hand, the intensity at a distance $r$ from the source is $I = \dfrac{P}{4\pi r^2}$, where $P$ is the power of the source. A fixed $P$ implies that $I_i r_i^2 = I_f r_f^2$. Thus, with $r_i = 1.2 \text{ m},$ we obtain

$$r_f = \left(\frac{I_i}{I_f}\right)^{1/2} r_i = \left(\frac{1}{10}\right)^{1/4} (1.2 \text{ m}) = 0.67 \text{ m}.$$

35. (a) The intensity is

$$I = \frac{P}{4\pi r^2} = \frac{30.0\,\text{W}}{(4\pi)(200\,\text{m})^2} = 5.97\times10^{-5}\ \text{W/m}^2.$$

(b) Let $A$ (= 0.750 cm$^2$) be the cross-sectional area of the microphone. Then the power intercepted by the microphone is

$$P' = IA = 0 = (6.0\times10^{-5}\ \text{W/m}^2)(0.750\,\text{cm}^2)(10^{-4}\ \text{m}^2/\text{cm}^2) = 4.48\times10^{-9}\ \text{W}.$$

36. Combining Eqs.17-28 and 17-29 we have $\beta = 10\log\left(\frac{P}{I_o 4\pi r^2}\right)$. Taking differences (for sounds *A* and *B*) we find

$$\Delta\beta = 10\log\left(\frac{P_A}{I_o 4\pi r^2}\right) - 10\log\left(\frac{P_B}{I_o 4\pi r^2}\right) = 10\log\left(\frac{P_A}{P_B}\right)$$

using well-known properties of logarithms. Thus, we see that $\Delta\beta$ is independent of *r* and can be evaluated anywhere.

(a) We can solve the above relation (once we know $\Delta\beta = 5.0$) for the ratio of powers; we find $P_A/P_B \approx 3.2$.

(b) At $r = 1000$ m it is easily seen (in the graph) that $\Delta\beta = 5.0$ dB. This is the same $\Delta\beta$ we expect to find, then, at $r = 10$ m.

37. (a) As discussed on page 408, the average potential energy transport rate is the same as that of the kinetic energy. This implies that the (average) rate for the total energy is

$$\left(\frac{dE}{dt}\right)_{\text{avg}} = 2\left(\frac{dK}{dt}\right)_{\text{avg}} = 2\left(\tfrac{1}{4}\rho A v \omega^2 s_m^2\right)$$

using Eq. 17-44. In this equation, we substitute $\rho = 1.21$ kg/m$^3$, $A = \pi r^2 = \pi(0.020 \text{ m})^2$, $v = 343$ m/s, $\omega = 3000$ rad/s, $s_m = 12 \times 10^{-9}$ m, and obtain the answer $3.4 \times 10^{-10}$ W.

(b) The second string is in a separate tube, so there is no question about the waves superposing. The total rate of energy, then, is just the addition of the two: $2(3.4 \times 10^{-10} \text{ W}) = 6.8 \times 10^{-10}$ W.

(c) Now we *do* have superposition, with $\phi = 0$, so the resultant amplitude is twice that of the individual wave which leads to the energy transport rate being four times that of part (a). We obtain $4(3.4 \times 10^{-10} \text{ W}) = 1.4 \times 10^{-9}$ W.

(d) In this case $\phi = 0.4\pi$, which means (using Eq. 17-39)

$$s_m' = 2\, s_m \cos(\phi/2) = 1.618 s_m .$$

This means the energy transport rate is $(1.618)^2 = 2.618$ times that of part (a). We obtain $2.618(3.4 \times 10^{-10} \text{ W}) = 8.8 \times 10^{-10}$ W.

(e) The situation is as shown in Fig. 17-14(b). The answer is zero.

38. (a) Using Eq. 17–39 with $v$ = 343 m/s and $n$ = 1, we find $f = nv/2L$ = 86 Hz for the fundamental frequency in a nasal passage of length $L$ = 2.0 m (subject to various assumptions about the nature of the passage as a “bent tube open at both ends”).

(b) The sound would be perceptible as *sound* (as opposed to just a general vibration) of very low frequency.

(c) Smaller $L$ implies larger $f$ by the formula cited above. Thus, the female's sound is of higher pitch (frequency).

39. (a) From Eq. 17–53, we have

$$f = \frac{nv}{2L} = \frac{(1)(250\,\text{m/s})}{2(0.150\,\text{m})} = 833\,\text{Hz}.$$

(b) The frequency of the wave on the string is the same as the frequency of the sound wave it produces during its vibration. Consequently, the wavelength in air is

$$\lambda = \frac{v_{\text{sound}}}{f} = \frac{348\,\text{m/s}}{833\,\text{Hz}} = 0.418\,\text{m}.$$

40. The distance between nodes referred to in the problem means that $\lambda/2 = 3.8$ cm, or $\lambda = 0.076$ m. Therefore, the frequency is

$$f = v/\lambda = (1500 \text{ m/s})/(0.076 \text{ m}) \approx 20 \times 10^3 \text{ Hz}.$$

41. (a) We note that 1.2 = 6/5. This suggests that both even and odd harmonics are present, which means the pipe is open at both ends (see Eq. 17-39).

(b) Here we observe 1.4 = 7/5. This suggests that only odd harmonics are present, which means the pipe is open at only one end (see Eq. 17-41).

42. At the beginning of the exercises and problems section in the textbook, we are told to assume $v_{sound}$ = 343 m/s unless told otherwise. The second harmonic of pipe $A$ is found from Eq. 17–39 with $n$ = 2 and $L = L_A$, and the third harmonic of pipe $B$ is found from Eq. 17–41 with $n$ = 3 and $L = L_B$. Since these frequencies are equal, we have

$$\frac{2v_{\text{sound}}}{2L_A} = \frac{3v_{\text{sound}}}{4L_B} \Rightarrow L_B = \frac{3}{4}L_A.$$

(a) Since the fundamental frequency for pipe $A$ is 300 Hz, we immediately know that the second harmonic has $f$ = 2(300 Hz) = 600 Hz. Using this, Eq. 17–39 gives

$$L_A = (2)(343 \text{ m/s})/2(600 \text{ s}^{-1}) = 0.572 \text{ m}.$$

(b) The length of pipe $B$ is $L_B = \frac{3}{4}L_A = 0.429\,\text{m}$.

43. (a) When the string (fixed at both ends) is vibrating at its lowest resonant frequency, exactly one-half of a wavelength fits between the ends. Thus, $\lambda = 2L$. We obtain

$$v = f\lambda = 2Lf = 2(0.220\text{ m})(920\text{ Hz}) = 405\text{ m/s}.$$

(b) The wave speed is given by $v = \sqrt{\tau/\mu}$, where $\tau$ is the tension in the string and $\mu$ is the linear mass density of the string. If $M$ is the mass of the (uniform) string, then $\mu = M/L$. Thus,

$$\tau = \mu v^2 = (M/L)v^2 = [(800 \times 10^{-6}\text{ kg})/(0.220\text{ m})]\,(405\text{ m/s})^2 = 596\text{ N}.$$

(c) The wavelength is $\lambda = 2L = 2(0.220\text{ m}) = 0.440\text{ m}$.

(d) The frequency of the sound wave in air is the same as the frequency of oscillation of the string. The wavelength is different because the wave speed is different. If $v_a$ is the speed of sound in air the wavelength in air is

$$\lambda_a = v_a/f = (343\text{ m/s})/(920\text{ Hz}) = 0.373\text{ m}.$$

44. The frequency is $f$ = 686 Hz and the speed of sound is $v_{\text{sound}}$ = 343 m/s. If $L$ is the length of the air-column, then using Eq. 17–41, the water height is (in unit of meters)

$$h = 1.00 - L = 1.00 - \frac{nv}{4f} = 1.00 - \frac{n(343)}{4(686)} = (1.00 - 0.125n)\text{ m}$$

where $n$ = 1, 3, 5,… with only one end closed.

(a) There are 4 values of $n$ ($n$ = 1,3,5,7) which satisfies $h > 0$.

(b) The smallest water height for resonance to occur corresponds to $n$ = 7 with $h = 0.125\text{ m}$.

(c) The second smallest water height corresponds to $n$ = 5 with $h$ = 0.375 m.

45. (a) Since the pipe is open at both ends there are displacement antinodes at both ends and an integer number of half-wavelengths fit into the length of the pipe. If $L$ is the pipe length and $\lambda$ is the wavelength then $\lambda = 2L/n$, where $n$ is an integer. If $v$ is the speed of sound then the resonant frequencies are given by $f = v/\lambda = nv/2L$. Now $L = 0.457$ m, so

$$f = n(344 \text{ m/s})/2(0.457 \text{ m}) = 376.4n \text{ Hz}.$$

To find the resonant frequencies that lie between 1000 Hz and 2000 Hz, first set $f = 1000$ Hz and solve for $n$, then set $f = 2000$ Hz and again solve for $n$. The results are 2.66 and 5.32, which imply that $n = 3$, 4, and 5 are the appropriate values of $n$. Thus, there are 3 frequencies.

(b) The lowest frequency at which resonance occurs is ($n = 3$) $f = 3(376.4 \text{ Hz}) = 1129$ Hz.

(c) The second lowest frequency at which resonance occurs is ($n = 4$)

$$f = 4(376.4 \text{ Hz}) = 1506 \text{ Hz}.$$

46. (a) Since the difference between consecutive harmonics is equal to the fundamental frequency (see section 17-6) then $f_1$ = (390 – 325) Hz = 65 Hz. The next harmonic after 195 Hz is therefore (195 + 65) Hz = 260 Hz.

(b) Since $f_n = nf_1$ then $n$ = 260/65 = 4.

(c) Only *odd* harmonics are present in tube *B* so the difference between consecutive harmonics is equal to *twice* the fundamental frequency in this case (consider taking differences of Eq. 17-41 for various values of *n*). Therefore,

$$f_1 = \frac{1}{2}(1320 - 1080)\text{ Hz} = 120\text{ Hz}.$$

The next harmonic after 600 Hz is consequently [600 + 2(120)] Hz = 840 Hz.

(d) Since $f_n = nf_1$ (for *n* odd) then $n$ = 840/120 = 7.

47. The string is fixed at both ends so the resonant wavelengths are given by $\lambda = 2L/n$, where $L$ is the length of the string and $n$ is an integer. The resonant frequencies are given by $f = v/\lambda = nv/2L$, where $v$ is the wave speed on the string. Now $v = \sqrt{\tau/\mu}$, where $\tau$ is the tension in the string and $\mu$ is the linear mass density of the string. Thus $f = (n/2L)\sqrt{\tau/\mu}$. Suppose the lower frequency is associated with $n = n_1$ and the higher frequency is associated with $n = n_1 + 1$. There are no resonant frequencies between so you know that the integers associated with the given frequencies differ by 1. Thus $f_1 = (n_1/2L)\sqrt{\tau/\mu}$ and

$$f_2 = \frac{n_1+1}{2L}\sqrt{\frac{\tau}{\mu}} = \frac{n_1}{2L}\sqrt{\frac{\tau}{\mu}} + \frac{1}{2L}\sqrt{\frac{\tau}{\mu}} = f_1 + \frac{1}{2L}\sqrt{\frac{\tau}{\mu}}.$$

This means $f_2 - f_1 = (1/2L)\sqrt{\tau/\mu}$ and

$$\tau = 4L^2\mu(f_2 - f_1)^2 = 4(0.300\,\text{m})^2(0.650\times10^{-3}\,\text{kg/m})(1320\,\text{Hz} - 880\,\text{Hz})^2 = 45.3\,\text{N}.$$

48. (a) Using Eq. 17–39 with $n$ = 1 (for the fundamental mode of vibration) and 343 m/s for the speed of sound, we obtain

$$f = \frac{(1)v_{\text{sound}}}{4L_{\text{tube}}} = \frac{343\,\text{m/s}}{4(1.20\,\text{m})} = 71.5\,\text{Hz}.$$

(b) For the wire (using Eq. 17–53) we have

$$f' = \frac{nv_{\text{wire}}}{2L_{\text{wire}}} = \frac{1}{2L_{\text{wire}}}\sqrt{\frac{\tau}{\mu}}$$

where $\mu = m_{\text{wire}}/L_{\text{wire}}$. Recognizing that $f = f'$ (both the wire and the air in the tube vibrate at the same frequency), we solve this for the tension $\tau$:

$$\tau = (2L_{\text{wire}}\, f)^2 \left(\frac{m_{\text{wire}}}{L_{\text{wire}}}\right) = 4f^2 m_{\text{wire}} L_{\text{wire}} = 4(71.5\,\text{Hz})^2 (9.60\times10^{-3}\,\text{kg})(0.330\,\text{m}) = 64.8\,\text{N}.$$

49. The top of the water is a displacement node and the top of the well is a displacement anti-node. At the lowest resonant frequency exactly one-fourth of a wavelength fits into the depth of the well. If $d$ is the depth and $\lambda$ is the wavelength then $\lambda = 4d$. The frequency is $f = v/\lambda = v/4d$, where $v$ is the speed of sound. The speed of sound is given by $v = \sqrt{B/\rho}$, where $B$ is the bulk modulus and $\rho$ is the density of air in the well. Thus $f = (1/4d)\sqrt{B/\rho}$ and

$$d = \frac{1}{4f}\sqrt{\frac{B}{\rho}} = \frac{1}{4(7.00\,\text{Hz})}\sqrt{\frac{1.33\times10^5\ \text{Pa}}{1.10\,\text{kg/m}^3}} = 12.4\,\text{m}.$$

50. We observe that "third lowest … frequency" corresponds to harmonic number $n_A = 3$ for pipe *A* which is open at both ends. Also, "second lowest … frequency" corresponds to harmonic number $n_B = 3$ for pipe *B* which is closed at one end.

(a) Since the frequency of *B* matches the frequency of A, using Eqs. 17-39 and 17-41, we have

$$f_A = f_B \quad \Rightarrow \frac{3v}{2L_A} = \frac{3v}{4L_B}$$

which implies $L_B = L_A/2 = (1.20\text{ m})/2 = 0.60\text{ m}$. Using Eq. 17-40, the corresponding wavelength is

$$\lambda = \frac{4L_B}{3} = \frac{4(0.60\text{ m})}{3} = 0.80\text{ m}.$$

The change from node to anti-node requires a distance of $\lambda/4$ so that every increment of 0.20 m along the *x* axis involves a switch between node and anti-node. Since the closed end is a node, the next node appears at $x = 0.40$ m So there are 2 nodes. The situation corresponds to that illustrated in Fig. 17-15(b) with $n = 3$.

(b) The smallest value of *x* where a node is present is $x = 0$.

(c) The second smallest value of *x* where a node is present is $x = 0.40$m.

(d) Using $v = 343$ m/s, we find $f_3 = v/\lambda = 429$ Hz. Now, we find the fundamental resonant frequency by dividing by the harmonic number, $f_1 = f_3/3 = 143$ Hz.

51. Let the period be $T$. Then the beat frequency is $1/T - 440\,\text{Hz} = 4.00\,\text{beats/s}$. Therefore, $T = 2.25 \times 10^{-3}$ s. The string that is "too tightly stretched" has the higher tension and thus the higher (fundamental) frequency.

52. Since the beat frequency equals the difference between the frequencies of the two tuning forks, the frequency of the first fork is either 381 Hz or 387 Hz. When mass is added to this fork its frequency decreases (recall, for example, that the frequency of a mass-spring oscillator is proportional to $1/\sqrt{m}$ ). Since the beat frequency also decreases the frequency of the first fork must be greater than the frequency of the second. It must be 387 Hz.

53. Each wire is vibrating in its fundamental mode so the wavelength is twice the length of the wire ($\lambda = 2L$) and the frequency is

$$f = v/\lambda = (1/2L)\sqrt{\tau/\mu},$$

where $v = \sqrt{\tau/\mu}$ is the wave speed for the wire, $\tau$ is the tension in the wire, and $\mu$ is the linear mass density of the wire. Suppose the tension in one wire is $\tau$ and the oscillation frequency of that wire is $f_1$. The tension in the other wire is $\tau + \Delta\tau$ and its frequency is $f_2$. You want to calculate $\Delta\tau/\tau$ for $f_1$ = 600 Hz and $f_2$ = 606 Hz. Now, $f_1 = (1/2L)\sqrt{\tau/\mu}$ and $f_2 = (1/2L)\sqrt{(\tau+\Delta\tau/\mu}$, so

$$f_2/f_1 = \sqrt{(\tau+\Delta\tau)/\tau} = \sqrt{1+(\Delta\tau/\tau)}.$$

This leads to $\Delta\tau/\tau = (f_2/f_1)^2 - 1 = [(606\,\text{Hz})/(600\,\text{Hz})]^2 - 1 = 0.020.$

54. (a) The number of different ways of picking up a pair of tuning forks out of a set of five is 5!/(2!3!) = 10. For each of the pairs selected, there will be one beat frequency. If these frequencies are all different from each other, we get the maximum possible number of 10.

(b) First, we note that the minimum number occurs when the frequencies of these forks, labeled 1 through 5, increase in equal increments: $f_n = f_1 + n\Delta f$, where $n$ = 2, 3, 4, 5. Now, there are only 4 different beat frequencies: $f_{\text{beat}} = n\Delta f$, where $n$ = 1, 2, 3, 4.

55. In the general Doppler shift equation, the trooper's speed is the source speed and the speeder's speed is the detector's speed. The Doppler effect formula, Eq. 17–47, and its accompanying rule for choosing ± signs, are discussed in §17-10. Using that notation, we have $v$ = 343 m/s,

$$v_D = v_S = 160 \text{ km/h} = (160000 \text{ m})/(3600 \text{ s}) = 44.4 \text{ m/s},$$

and $f$ = 500 Hz. Thus,

$$f' = (500 \text{ Hz})\left(\frac{343 \text{ m/s} - 44.4 \text{ m/s}}{343 \text{ m/s} - 44.4 \text{ m/s}}\right) = 500\,\text{Hz} \;\Rightarrow\; \Delta f = 0.$$

56. The Doppler effect formula, Eq. 17–47, and its accompanying rule for choosing ± signs, are discussed in §17-10. Using that notation, we have $v$ = 343 m/s, $v_D$ = 2.44 m/s, $f'$ = 1590 Hz and $f$ = 1600 Hz. Thus,

$$f' = f\left(\frac{v+v_D}{v+v_S}\right) \Rightarrow v_S = \frac{f}{f'}\,(v+v_D) - v = 4.61\,\text{m/s}.$$

57. We use $v_S = r\omega$ (with $r$ = 0.600 m and $\omega$ = 15.0 rad/s) for the linear speed during circular motion, and Eq. 17–47 for the Doppler effect (where $f$ = 540 Hz, and $v$ = 343 m/s for the speed of sound).

(a) The lowest frequency is

$$f' = f\left(\frac{v+0}{v+v_S}\right) = 526 \text{ Hz}.$$

(b) The highest frequency is

$$f' = f\left(\frac{v+0}{v-v_S}\right) = 555 \text{ Hz}.$$

58. We are combining two effects: the reception of a moving object (the truck of speed $u$ = 45.0 m/s) of waves emitted by a stationary object (the motion detector), and the subsequent emission of those waves by the moving object (the truck) which are picked up by the stationary detector. This could be figured in two steps, but is more compactly computed in one step as shown here:

$$f_{\text{final}} = f_{\text{initial}}\left(\frac{v+u}{v-u}\right) = (0.150\,\text{MHz})\left(\frac{343\,\text{m/s} + 45\,\text{m/s}}{343\,\text{m/s} - 45\,\text{m/s}}\right) = 0.195\,\text{MHz}.$$

59. In this case, the intruder is moving *away* from the source with a speed $u$ satisfying $u/v \ll 1$. The Doppler shift (with $u$ = –0.950 m/s) leads to

$$f_{\text{beat}} = \left| f_r - f_s \right| \approx \frac{2\,|\,u\,|}{v}\, f_s = \frac{2(0.95\,\text{m/s})(28.0\text{ kHz})}{343\,\text{m/s}}) = 155\,\text{Hz}\,.$$

60. We use Eq. 17–47 with $f$ = 1200 Hz and $v$ = 329 m/s.

(a) In this case, $v_D$ = 65.8 m/s and $v_S$ = 29.9 m/s, and we choose signs so that $f'$ is larger than $f$:

$$f' = f\left(\frac{329\text{ m/s} + 65.8\text{ m/s}}{329\text{ m/s} - 29.9\text{ m/s}}\right) = 1.58\times 10^3\text{ Hz.}$$

(b) The wavelength is $\lambda = v/f'$ = 0.208 m.

(c) The wave (of frequency $f'$) "emitted" by the moving reflector (now treated as a "source," so $v_S$ = 65.8 m/s) is returned to the detector (now treated as a detector, so $v_D$ = 29.9 m/s) and registered as a new frequency $f''$:

$$f'' = f'\left(\frac{329\text{ m/s} + 29.9\text{ m/s}}{329\text{ m/s} - 65.8\text{ m/s}}\right) = 2.16\times 10^3\text{ Hz.}$$

(d) This has wavelength $v/f''$ = 0.152 m.

61. We denote the speed of the French submarine by $u_1$ and that of the U.S. sub by $u_2$.

(a) The frequency as detected by the U.S. sub is

$$f_1' = f_1\left(\frac{v+u_2}{v-u_1}\right) = (1.000\times10^3\ \text{Hz})\left(\frac{5470\ \text{km/h}\ +70.00\ \text{km/h}}{5470\ \text{km/h}\ -\ 50.00\ \text{km/h}}\right) = 1.022\ \times\ 10^3\ \text{Hz}.$$

(b) If the French sub were stationary, the frequency of the reflected wave would be $f_r = f_1(v+u_2)/(v - u_2)$. Since the French sub is moving towards the reflected signal with speed $u_1$, then

$$f_r' = f_r\left(\frac{v+u_1}{v}\right) = f_1\frac{(v+u_1)(v+u_2)}{v(v-u_2)} = \frac{(1.000\times10^3\ \text{Hz})(5470+50.00)(5470+70.00)}{(5470)(5470-70.00)}$$

$$= 1.045\times10^3\ \text{Hz}.$$

62. When the detector is stationary (with respect to the air) then Eq. 17-47 gives

$$f' = \frac{f}{1 - v_s / v}$$

where $v_s$ is the speed of the source (assumed to be approaching the detector in the way we've written it, above). The difference between the approach and the recession is

$$f' - f'' = f\left(\frac{1}{1 - v_s/v} - \frac{1}{1 + v_s/v}\right) = f\left(\frac{2\, v_s/v}{1 - (v_s/v)^2}\right)$$

which, after setting $(f' - f'')/f = 1/2$, leads to an equation which can be solved for the ratio $v_s/v$. The result is $\sqrt{5} - 2 = 0.236$. Thus, $v_s/v = 0.236$.

63. As a result of the Doppler effect, the frequency of the reflected sound as heard by the bat is

$$f_r = f'\left(\frac{v+u_{\text{bat}}}{v-u_{\text{bat}}}\right) = (3.9\times10^4\ \text{Hz})\left(\frac{v+v/40}{v-v/40}\right) = 4.1\times10^4\ \text{Hz}.$$

64. The "third harmonic" refers to a resonant frequency $f_3 = 3\ f_1$, where $f_1$ is the fundamental lowest resonant frequency. When the source is stationary, with respect to the air, then Eq. 17-47 gives

$$f' = f\left(1 - \frac{v_d}{v}\right)$$

where $v_d$ is the speed of the detector (assumed to be moving away from the source, in the way we've written it, above). The problem, then, wants us to find $v_d$ such that $f' = f_1$ when the emitted frequency is $f = f_3$. That is, we require $1 - v_d/v = 1/3$. Clearly, the solution to this is $v_d/v = 2/3$ (independent of length and whether one or both ends are open [the latter point being due to the fact that the odd harmonics occur in both systems]). Thus,

(a) For tube 1, $v_d = 2v/3$.

(b) For tube 2, $v_d = 2v/3$.

(c) For tube 3, $v_d = 2v/3$.

(d) For tube 4, $v_d = 2v/3$.

65. (a) The expression for the Doppler shifted frequency is

$$f' = f\,\frac{v \pm v_D}{v \mp v_S},$$

where $f$ is the unshifted frequency, $v$ is the speed of sound, $v_D$ is the speed of the detector (the uncle), and $v_S$ is the speed of the source (the locomotive). All speeds are relative to the air. The uncle is at rest with respect to the air, so $v_D = 0$. The speed of the source is $v_S = 10$ m/s. Since the locomotive is moving away from the uncle the frequency decreases and we use the plus sign in the denominator. Thus

$$f' = f\,\frac{v}{v + v_S} = (500.0\,\text{Hz})\left(\frac{343\,\text{m/s}}{343\,\text{m/s} + 10.00\,\text{m/s}}\right) = 485.8\,\text{Hz}.$$

(b) The girl is now the detector. Relative to the air she is moving with speed $v_D = 10.00$ m/s toward the source. This tends to increase the frequency and we use the plus sign in the numerator. The source is moving at $v_S = 10.00$ m/s away from the girl. This tends to decrease the frequency and we use the plus sign in the denominator. Thus $(v + v_D) = (v + v_S)$ and $f' = f = 500.0$ Hz.

(c) Relative to the air the locomotive is moving at $v_S = 20.00$ m/s away from the uncle. Use the plus sign in the denominator. Relative to the air the uncle is moving at $v_D = 10.00$ m/s toward the locomotive. Use the plus sign in the numerator. Thus

$$f' = f\,\frac{v + v_D}{v + v_S} = (500.0\,\text{Hz})\left(\frac{343\,\text{m/s} + 10.00\,\text{m/s}}{343\,\text{m/s} + 20.00\,\text{m/s}}\right) = 486.2\,\text{Hz}.$$

(d) Relative to the air the locomotive is moving at $v_S = 20.00$ m/s away from the girl and the girl is moving at $v_D = 20.00$ m/s toward the locomotive. Use the plus signs in both the numerator and the denominator. Thus $(v + v_D) = (v + v_S)$ and $f' = f = 500.0$ Hz.

66. We use Eq. 17–47 with $f$ = 500 Hz and $v$ = 343 m/s. We choose signs to produce $f' > f$.

(a) The frequency heard in still air is

$$f' = (500\ \text{Hz})\left(\frac{343\ \text{m/s} + 30.5\ \text{m/s}}{343\ \text{m/s} - 30.5\ \text{m/s}}\right) = 598\,\text{Hz}.$$

(b) In a frame of reference where the air seems still, the velocity of the detector is 30.5 – 30.5 = 0, and that of the source is 2(30.5). Therefore,

$$f' = (500\ \text{Hz})\left(\frac{343\ \text{m/s} + 0}{343\ \text{m/s} - 2(30.5\ \text{m/s})}\right) = 608\,\text{Hz}.$$

(c) We again pick a frame of reference where the air seems still. Now, the velocity of the source is 30.5 – 30.5 = 0, and that of the detector is 2(30.5). Consequently,

$$f' = (500\ \text{Hz})\left(\frac{343\ \text{m/s} + 2(30.5\ \text{m/s})}{343\ \text{m/s} - 0}\right) = 589\,\text{Hz}.$$

67. The Doppler shift formula, Eq. 17–47, is valid only when both $u_S$ and $u_D$ are measured with respect to a stationary medium (i.e., no wind). To modify this formula in the presence of a wind, we switch to a new reference frame in which there is no wind.

(a) When the wind is blowing from the source to the observer with a speed $w$, we have $u'_S = u'_D = w$ in the new reference frame that moves together with the wind. Since the observer is now approaching the source while the source is backing off from the observer, we have, in the new reference frame,

$$f' = f\left(\frac{v+u'_D}{v+u'_S}\right) = f\left(\frac{v+w}{v+w}\right) = 2.0\times10^3 \text{ Hz}.$$

In other words, there is no Doppler shift.

(b) In this case, all we need to do is to reverse the signs in front of both $u'_D$ and $u'_S$. The result is that there is still no Doppler shift:

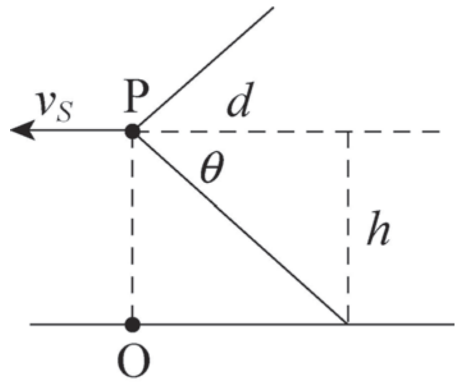
$$f' = f\left(\frac{v-u'_D}{v-u'_S}\right) = f\left(\frac{v-w}{v-w}\right) = 2.0\times10^3 \text{ Hz}.$$

In general, there will always be no Doppler shift as long as there is no relative motion between the observer and the source, regardless of whether a wind is present or not.

68. We note that 1350 km/h is $v_S$ = 375 m/s. Then, with $\theta = 60^{\mathrm{o}}$, Eq. 17-57 gives $v$ = $3.3\times10^2$ m/s.

69. (a) The half angle $\theta$ of the Mach cone is given by sin $\theta = v/v_S$, where $v$ is the speed of sound and $v_S$ is the speed of the plane. Since $v_S = 1.5v$, sin $\theta = v/1.5v = 1/1.5$. This means $\theta = 42°$.

(b) Let $h$ be the altitude of the plane and suppose the Mach cone intersects Earth's surface a distance $d$ behind the plane. The situation is shown on the diagram below, with P indicating the plane and O indicating the observer. The cone angle is related to $h$ and $d$ by tan $\theta = h/d$, so $d = h/\tan\theta$. The shock wave reaches O in the time the plane takes to fly the distance $d$:




$$t = \frac{d}{v} = \frac{h}{v\tan\theta} = \frac{5000 \text{ m}}{1.5(331 \text{ m/s})\tan 42°} = 11 \text{ s}.$$

70. The altitude *H* and the horizontal distance *x* for the legs of a right triangle, so we have

$$H = x\tan\theta = v_p t\tan\theta = 1.25vt\sin\theta$$

where *v* is the speed of sound, $v_p$ is the speed of the plane and

$$\theta = \sin^{-1}\left(\frac{v}{v_p}\right) = \sin^{-1}\left(\frac{v}{1.25v}\right) = 53.1°.$$

Thus the altitude is

$$H = x\tan\theta = (1.25)(330\,\text{m/s})(60\,\text{s})(\tan 53.1°) = 3.30\times10^4\ \text{m}.$$

71. (a) Incorporating a term ($\lambda/2$) to account for the phase shift upon reflection, then the path difference for the waves (when they come back together) is

$$\sqrt{L^2 + (2d)^2} - L + \lambda/2 = \Delta(\text{path}) .$$

Setting this equal to the condition needed to destructive interference ($\lambda/2$, $3\lambda/2$, $5\lambda/2$ …) leads to $d$ = 0, 2.10 m, … Since the problem explicitly excludes the $d$ = 0 possibility, then our answer is $d$ = 2.10 m.

(b) Setting this equal to the condition needed to constructive interference ($\lambda$, $2\lambda$, $3\lambda$ …) leads to $d$ = 1.47 m, … Our answer is $d$ = 1.47 m.

72. When the source is stationary (with respect to the air) then Eq. 17-47 gives

$$f' = f\left(1 - \frac{v_d}{v}\right),$$

where $v_d$ is the speed of the detector (assumed to be moving away from the source, in the way we've written it, above). The difference between the approach and the recession is

$$f'' - f' = f\left[\left(1 + \frac{v_d}{v}\right) - \left(1 - \frac{v_d}{v}\right)\right] = f\left(2\frac{v_d}{v}\right)$$

which, after setting $(f'' - f')/f = 1/2$, leads to an equation which can be solved for the ratio $v_d/v$. The result is 1/4. Thus, $v_d/v = 0.250$.

73. (a) Adapting Eq. 17-39 to the notation of this chapter, we have

$$s_m' = 2\, s_m \cos(\phi/2) = 2(12 \text{ nm}) \cos(\pi/6) = 20.78 \text{ nm}.$$

Thus, the amplitude of the resultant wave is roughly 21 nm.

(b) The wavelength ($\lambda$ = 35 cm) does not change as a result of the superposition.

(c) Recalling Eq. 17-47 (and the accompanying discussion) from the previous chapter, we conclude that the standing wave amplitude is 2(12 nm) = 24 nm when they are traveling in opposite directions.

(d) Again, the wavelength ($\lambda$ = 35 cm) does not change as a result of the superposition.

74. (a) The separation distance between points *A* and *B* is one-quarter of a wavelength; therefore, $\lambda = 4(0.15\text{ m}) = 0.60\text{ m}$. The frequency, then, is

$$f = v/\lambda = (343\text{ m/s})/(0.60\text{ m}) = 572\text{ Hz}.$$

(b) The separation distance between points *C* and *D* is one-half of a wavelength; therefore, $\lambda = 2(0.15\text{ m}) = 0.30\text{ m}$. The frequency, then, is

$$f = v/\lambda = (343\text{ m/s})/(0.30\text{ m}) = 1144\text{ Hz (or approximately 1.14 kHz)}.$$

75. Any phase changes associated with the reflections themselves are rendered inconsequential by the fact that there are an even number of reflections. The additional path length traveled by wave $A$ consists of the vertical legs in the zig-zag path: $2L$. To be (minimally) out of phase means, therefore, that $2L = \lambda/2$ (corresponding to a half-cycle, or 180°, phase difference). Thus, $L = \lambda/4$, or $L/\lambda = 1/4 = 0.25$.

76. Since they are approaching each other, the sound produced (of emitted frequency $f$) by the flatcar-trumpet received by an observer on the ground will be of higher pitch $f'$. In these terms, we are told $f' - f = 4.0$ Hz, and consequently that $f'/f = 444/440 = 1.0091$. With $v_S$ designating the speed of the flatcar and $v = 343$ m/s being the speed of sound, the Doppler equation leads to

$$\frac{f'}{f} = \frac{v+0}{v-v_S} \;\Rightarrow\; v_S = (343 \text{ m/s})\frac{1.0091-1}{1.0091} = 3.1\text{m/s}.$$

77. The siren is between you and the cliff, moving away from you and towards the cliff. Both "detectors" (you and the cliff) are stationary, so $v_D = 0$ in Eq. 17–47 (and see the discussion in the textbook immediately after that equation regarding the selection of ± signs). The source is the siren with $v_S = 10$ m/s. The problem asks us to use $v = 330$ m/s for the speed of sound.

(a) With $f = 1000$ Hz, the frequency $f_y$ you hear becomes

$$f_y = f\left(\frac{v+0}{v+v_S}\right) = 970.6\,\text{Hz} \approx 9.7\times10^2\ \text{Hz}.$$

(b) The frequency heard by an observer at the cliff (and thus the frequency of the sound reflected by the cliff, ultimately reaching your ears at some distance from the cliff) is

$$f_c = f\left(\frac{v+0}{v-v_S}\right) = 1031.3\,\text{Hz} \approx 1.0\times10^3\ \text{Hz}.$$

(c) The beat frequency is $f_c - f_y = 60$ beats/s (which, due to specific features of the human ear, is too large to be perceptible).

78. Let $r$ stand for the ratio of the source speed to the speed of sound. Then, Eq. 17-55 (plus the fact that frequency is inversely proportional to wavelength) leads to

$$2\left(\frac{1}{1+r}\right) = \frac{1}{1-r} \ .$$

Solving, we find $r = 1/3$. Thus, $v_s/v = 0.33$.

79. The source being isotropic means $A_{\text{sphere}} = 4\pi r^2$ is used in the intensity definition $I = P/A$, which further implies

$$\frac{I_2}{I_1} = \frac{P/4\pi r_2^2}{P/4\pi r_1^2} = \left(\frac{r_1}{r_2}\right)^2 .$$

(a) With $I_1 = 9.60 \times 10^{-4}$ W/m$^2$, $r_1 = 6.10$ m, and $r_2 = 30.0$ m, we find

$$I_2 = (9.60 \times 10^{-4} \text{ W/m}^2)(6.10/30.0)^2 = 3.97 \times 10^{-5} \text{ W/m}^2.$$

(b) Using Eq. 17–27 with $I_1 = 9.60 \times 10^{-4}$ W/m$^2$, $\omega = 2\pi(2000$ Hz), $v = 343$ m/s and $\rho = 1.21$ kg/m$^3$, we obtain

$$s_m = \sqrt{\frac{2I}{\rho v \omega^2}} = 1.71 \times 10^{-7} \text{ m}.$$

(c) Eq. 17-15 gives the pressure amplitude:

$$\Delta p_m = \rho v \omega s_m = 0.893 \text{ Pa}.$$

80. When $\phi = 0$ it is clear that the superposition wave has amplitude $2\Delta p_m$. For the other cases, it is useful to write

$$\Delta p_1 + \Delta p_2 = \Delta p_m \left(\sin(\omega t) + \sin(\omega t - \phi)\right) = \left(2\Delta p_m \cos\frac{\phi}{2}\right)\sin\left(\omega t - \frac{\phi}{2}\right).$$

The factor in front of the sine function gives the amplitude $\Delta p_r$. Thus, $\Delta p_r / \Delta p_m = 2\cos(\phi/2).$

(a) When $\phi = 0$, $\Delta p_r / \Delta p_m = 2\cos(0) = 2.00.$

(b) When $\phi = \pi/2$, $\Delta p_r / \Delta p_m = 2\cos(\pi/4) = \sqrt{2} = 1.41.$

(c) When $\phi = \pi/3$, $\Delta p_r / \Delta p_m = 2\cos(\pi/6) = \sqrt{3} = 1.73.$

(d) When $\phi = \pi/4$, $\Delta p_r / \Delta p_m = 2\cos(\pi/8) = 1.85.$

81. (a) With $r$ = 10 m in Eq. 17–28, we have

$$I = \frac{P}{4\pi r^2} \quad \Rightarrow \quad P = 10\,\text{W}.$$

(b) Using that value of $P$ in Eq. 17–28 with a new value for $r$, we obtain

$$I = \frac{P}{4\pi(5.0)^2} = 0.032\,\frac{\text{W}}{\text{m}^2}.$$

Alternatively, a ratio $I'/I = (r/r')^2$ could have been used.

(c) Using Eq. 17–29 with $I = 0.0080\ \text{W/m}^2$, we have

$$\beta = 10\log\frac{I}{I_0} = 99\,\text{dB}$$

where $I_0 = 1.0 \times 10^{-12}\ \text{W/m}^2$.

82. We use $v = \sqrt{B/\rho}$ to find the bulk modulus $B$:

$$B = v^2\rho = \left(5.4\times10^3\ \text{m/s}\right)^2\left(2.7\times10^3\ \text{kg/m}^3\right) = 7.9\times10^{10}\ \text{Pa}.$$

83. Let the frequencies of sound heard by the person from the left and right forks be $f_l$ and $f_r$, respectively.

(a) If the speeds of both forks are $u$, then $f_{l,r} = fv/(v \pm u)$ and

$$f_{\text{beat}} = |f_r - f_l| = fv\left(\frac{1}{v-u} - \frac{1}{v+u}\right) = \frac{2\,fuv}{v^2 - u^2} = \frac{2(440\,\text{Hz})(3.00\,\text{m/s})(343\,\text{m/s})}{(343\,\text{m/s})^2 - (3.00\,\text{m/s})^2}$$

$$= 7.70\,\text{Hz}.$$

(b) If the speed of the listener is $u$, then $f_{l,r} = f(v \pm u)/v$ and

$$f_{\text{beat}} = |f_l - f_r| = 2f\left(\frac{u}{v}\right) = 2(440\,\text{Hz})\left(\frac{3.00\,\text{m/s}}{343\,\text{m/s}}\right) = 7.70\,\text{Hz}.$$

84. The rule: if you divide the time (in seconds) by 3, then you get (approximately) the straight-line distance $d$. We note that the speed of sound we are to use is given at the beginning of the problem section in the textbook, and that the speed of light is very much larger than the speed of sound. The proof of our rule is as follows:

$$t = t_{\text{sound}} - t_{\text{light}} \approx t_{\text{sound}} = \frac{d}{v_{\text{sound}}} = \frac{d}{343\,\text{m/s}} = \frac{d}{0.343\,\text{km/s}}.$$

Cross-multiplying yields (approximately) (0.3 km/s)$t$ = $d$ which (since 1/3 ≈ 0.3) demonstrates why the rule works fairly well.

85. (a) The intensity is given by $I = \frac{1}{2}\rho v\omega^2 s_m^2$, where $\rho$ is the density of the medium, $v$ is the speed of sound, $\omega$ is the angular frequency, and $s_m$ is the displacement amplitude. The displacement and pressure amplitudes are related by $\Delta p_m = \rho v\omega s_m$, so $s_m = \Delta p_m/\rho v\omega$ and $I = (\Delta p_m)^2/2\rho v$. For waves of the same frequency the ratio of the intensity for propagation in water to the intensity for propagation in air is

$$\frac{I_w}{I_a} = \left(\frac{\Delta p_{mw}}{\Delta p_{ma}}\right)^2 \frac{\rho_a v_a}{\rho_w v_w},$$

where the subscript $a$ denotes air and the subscript $w$ denotes water. Since $I_a = I_w$,

$$\frac{\Delta p_{mw}}{\Delta p_{ma}} = \sqrt{\frac{\rho_w v_w}{\rho_a v_a}} = \sqrt{\frac{(0.998\times10^3\ \text{kg/m}^3)(1482\ \text{m/s})}{(1.21\ \text{kg/m}^3)(343\ \text{m/s})}} = 59.7.$$

The speeds of sound are given in Table 17-1 and the densities are given in Table 15-1.

(b) Now, $\Delta p_{mw} = \Delta p_{ma}$, so

$$\frac{I_w}{I_a} = \frac{\rho_a v_a}{\rho_w v_w} = \frac{(1.21\ \text{kg/m}^3)(343\ \text{m/s})}{(0.998\times10^3\ \text{kg/m}^3)(1482\ \text{m/s})} = 2.81\times10^{-4}.$$

86. We use $\Delta\beta_{12} = \beta_1 - \beta_2 = (10\text{ dB})\log(I_1/I_2)$.

(a) Since $\Delta\beta_{12} = (10\text{ dB})\log(I_1/I_2) = 37\text{ dB}$, we get

$$I_1/I_2 = 10^{37\text{ dB}/10\text{ dB}} = 10^{3.7} = 5.0 \times 10^3.$$

(b) Since $\Delta p_m \propto s_m \propto \sqrt{I}$, we have

$$\Delta p_{m1}/\Delta p_{m2} = \sqrt{I_1/I_2} = \sqrt{5.0\times10^3} = 71.$$

(c) The displacement amplitude ratio is $s_{m1}/s_{m2} = \sqrt{I_1/I_2} = 71$.

87. (a) When the right side of the instrument is pulled out a distance *d* the path length for sound waves increases by 2*d*. Since the interference pattern changes from a minimum to the next maximum, this distance must be half a wavelength of the sound. So $2d = \lambda/2$, where $\lambda$ is the wavelength. Thus $\lambda = 4d$ and, if *v* is the speed of sound, the frequency is

$$f = v/\lambda = v/4d = (343 \text{ m/s})/4(0.0165 \text{ m}) = 5.2 \times 10^3 \text{ Hz}.$$

(b) The displacement amplitude is proportional to the square root of the intensity (see Eq. 17–27). Write $\sqrt{I} = Cs_m$, where *I* is the intensity, $s_m$ is the displacement amplitude, and *C* is a constant of proportionality. At the minimum, interference is destructive and the displacement amplitude is the difference in the amplitudes of the individual waves: $s_m = s_{SAD} - s_{SBD}$, where the subscripts indicate the paths of the waves. At the maximum, the waves interfere constructively and the displacement amplitude is the sum of the amplitudes of the individual waves: $s_m = s_{SAD} + s_{SBD}$. Solve

$$\sqrt{100} = C(s_{SAD} - s_{SBD}) \quad \text{and} \quad \sqrt{900} = C(s_{SAD} - s_{SBD})$$

for $s_{SAD}$ and $s_{SBD}$. Adding the equations give

$$s_{SAD} = (\sqrt{100} + \sqrt{900} / 2C = 20 / C,$$

while subtracting them yields

$$s_{SBD} = (\sqrt{900} - \sqrt{100}) / 2C = 10 / C.$$

Thus, the ratio of the amplitudes is $s_{SAD}/s_{SBD} = 2$.

(c) Any energy losses, such as might be caused by frictional forces of the walls on the air in the tubes, result in a decrease in the displacement amplitude. Those losses are greater on path B since it is longer than path A.

88. The angle is $\sin^{-1}(v/v_s) = \sin^{-1}(343/685) = 30°$.

89. The round-trip time is $t = 2L/v$ where we estimate from the chart that the time between clicks is 3 ms. Thus, with $v$ = 1372 m/s, we find $L = \frac{1}{2}vt = 2.1\ \text{m}$.

90. The wave is written as $s(x,t) = s_m \cos(kx \pm \omega t)$.

(a) The amplitude $s_m$ is equal to the maximum displacement: $s_m = 0.30 \text{ cm}$.

(b) Since λ = 24 cm, the angular wave number is $k = 2\pi / \lambda = 0.26 \text{ cm}^{-1}$.

(c) The angular frequency is $\omega = 2\pi f = 2\pi(25 \text{ Hz}) = 1.6\times10^2 \text{ rad/s}$.

(d) The speed of the wave is $v = \lambda f$ = (24 cm)(25 Hz) = $6.0 \times 10^2$ cm/s.

(e) Since the direction of propagation is $-x$, the sign is plus, i.e., $s(x,t) = s_m \cos(kx + \omega t)$.

91. The source being a "point source" means $A_{\text{sphere}} = 4\pi r^2$ is used in the intensity definition $I = P/A$, which further implies

$$\frac{I_2}{I_1} = \frac{P/4\pi r_2^2}{P/4\pi r_1^2} = \left(\frac{r_1}{r_2}\right)^2 .$$

From the discussion in §17-5, we know that the intensity ratio between "barely audible" and the "painful threshold" is $10^{-12} = I_2/I_1$. Thus, with $r_2$ = 10000 m, we find

$$r_1 = r_2\sqrt{10^{-12}} = 0.01\,\text{m} = 1 \text{ cm}.$$

92. (a) The time it takes for sound to travel in air is $t_a = L/v$, while it takes $t_m = L/v_m$ for the sound to travel in the metal. Thus,

$$\Delta t = t_a - t_m = \frac{L}{v} - \frac{L}{v_m} = \frac{L(v_m - v)}{v_m v}.$$

(b) Using the values indicated (see Table 17-1), we obtain

$$L = \frac{\Delta t}{1/v - 1/v_m} = \frac{1.00\,\text{s}}{1/(343\,\text{m/s}) \; - \; 1/(5941\,\text{m/s})} = 364\,\text{m}.$$

93. (a) We observe that "third lowest … frequency" corresponds to harmonic number $n$ = 5 for such a system. Using Eq. 17–41, we have

$$f = \frac{nv}{4L} \quad \Rightarrow \quad 750\,\text{Hz} = \frac{5v}{4(0.60\text{ m})}$$

so that $v = 3.6\times10^2$ m/s.

(b) As noted, $n$ = 5; therefore, $f_1$ = 750/5 = 150 Hz.

94. We note that waves 1 and 3 differ in phase by $\pi$ radians (so they cancel upon superposition). Waves 2 and 4 also differ in phase by $\pi$ radians (and also cancel upon superposition). Consequently, there is no resultant wave.

95. Since they oscillate out of phase, then their waves will cancel (producing a node) at a point exactly midway between them (the midpoint of the system, where we choose $x = 0$). We note that Figure 17-14, and the $n = 3$ case of Figure 17-15(a) have this property (of a node at the midpoint). The distance $\Delta x$ between nodes is $\lambda/2$, where $\lambda = v/f$ and $f = 300$ Hz and $v = 343$ m/s. Thus, $\Delta x = v/2f = 0.572$ m.

Therefore, nodes are found at the following positions:

$$x = n\Delta x = n(0.572\,\text{m}),\ \ n = 0, \pm 1, \pm 2, \ldots$$

(a) The shortest distance from the midpoint where nodes are found is $\Delta x = 0$.

(b) The second shortest distance from the midpoint where nodes are found is $\Delta x = 0.572$ m.

(c) The third shortest distance from the midpoint where nodes are found is $2\Delta x = 1.14$ m.

96. (a) With $f$ = 686 Hz and $v$ = 343 m/s, then the "separation between adjacent wavefronts" is $\lambda = v/f$ = 0.50 m.

(b) This is one of the effects which are part of the Doppler phenomena. Here, the wavelength shift (relative to its "true" value in part (a)) equals the source speed $v_s$ (with appropriate $\pm$ sign) relative to the speed of sound $v$:

$$\frac{\Delta\lambda}{\lambda} = \pm\frac{v_s}{v}.$$

In front of the source, the shift in wavelength is –(0.50 m)(110 m/s)/(343 m/s) = –0.16 m, and the wavefront separation is 0.50 m – 0.16 m = 0.34 m.

(c) Behind the source, the shift in wavelength is +(0.50 m)(110 m/s)/(343 m/s) = +0.16 m, and the wavefront separation is 0.50 m + 0.16 m = 0.66 m.

97. We use $I \propto r^{-2}$ appropriate for an isotropic source. We have

$$\frac{I_{r=d}}{I_{r=D-d}} = \frac{(D-d)^2}{D^2} = \frac{1}{2},$$

where $d$ = 50.0 m. We solve for

$$D: D = \sqrt{2}d / \left(\sqrt{2} - 1\right) = \sqrt{2}\,(50.0\,\text{m}) / \left(\sqrt{2} - 1\right) = 171\,\text{m}.$$

98. (a) Using $m$ = 7.3 × $10^7$ kg, the initial gravitational potential energy is $U = mgy = 3.9\times10^{11}$ J, where $h$ = 550 m. Assuming this converts primarily into kinetic energy during the fall, then $K$ = 3.9 × $10^{11}$ J just before impact with the ground. Using instead the mass estimate $m$ = 1.7 × $10^8$ kg, we arrive at $K$ = 9.2 × $10^{11}$ J.

(b) The process of converting this kinetic energy into other forms of energy (during the impact with the ground) is assumed to take $\Delta t$ = 0.50 s (and in the average sense, we take the "power" $P$ to be wave-energy/$\Delta t$). With 20% of the energy going into creating a seismic wave, the intensity of the body wave is estimated to be

$$I = \frac{P}{A_{\text{hemisphere}}} = \frac{(0.20)K/\Delta t}{\frac{1}{2}(4\pi r^2)} = 0.63\,\text{W/m}^2$$

using $r$ = 200 × $10^3$ m and the smaller value for $K$ from part (a). Using instead the larger estimate for $K$, we obtain $I$ = 1.5 W/m$^2$.

(c) The surface area of a cylinder of "height" $d$ is $2\pi rd$, so the intensity of the surface wave is

$$I = \frac{P}{A_{\text{cylinder}}} = \frac{(0.20)K/\Delta t}{(2\pi rd)} = 25\times10^3\,\text{W/m}^2$$

using $d$ = 5.0 m, $r$ = 200 × $10^3$ m and the smaller value for $K$ from part (a). Using instead the larger estimate for $K$, we obtain $I$ = 58 kW/m$^2$.

(d) Although several factors are involved in determining which seismic waves are most likely to be detected, we observe that on the basis of the above findings we should expect the more intense waves (the surface waves) to be more readily detected.

99. (a) The period is the reciprocal of the frequency:

$$T = 1/f = 1/(90\ \text{Hz}) = 1.1 \times 10^{-2}\ \text{s}.$$

(b) Using $v$ = 343 m/s, we find $\lambda = v/f$ = 3.8 m.

100. (a) The problem asks for the source frequency $f$. We use Eq. 17–47 with great care (regarding its ± sign conventions).

$$f' = f\left(\frac{340\text{ m/s} - 16\text{ m/s}}{340\text{ m/s} - 40\text{ m/s}}\right)$$

Therefore, with $f'$ = 950 Hz, we obtain $f$ = 880 Hz.

(b) We now have

$$f' = f\left(\frac{340\text{ m/s} + 16\text{ m/s}}{340\text{ m/s} + 40\text{ m/s}}\right)$$

so that with $f$ = 880 Hz, we find $f'$ = 824 Hz.

101. (a) The blood is moving towards the right (towards the detector), because the Doppler shift in frequency is an *increase*: $\Delta f > 0$.

(b) The reception of the ultrasound by the blood and the subsequent remitting of the signal by the blood back toward the detector is a two-step process which may be compactly written as

$$f + \Delta f = f\left(\frac{v + v_x}{v - v_x}\right)$$

where $v_x = v_{\text{blood}} \cos\theta$. If we write the ratio of frequencies as $R = (f + \Delta f)/f$, then the solution of the above equation for the speed of the blood is

$$v_{\text{blood}} = \frac{(R-1)v}{(R+1)\cos\theta} = 0.90\,\text{m/s}$$

where $v$ = 1540 m/s, $\theta$ = 20°, and $R = 1 + 5495/5 \times 10^6$.

(c) We interpret the question as asking how $\Delta f$ (still taken to be positive, since the detector is in the "forward" direction) changes as the detection angle $\theta$ changes. Since larger $\theta$ means smaller horizontal component of velocity $v_x$ then we expect $\Delta f$ to decrease towards zero as $\theta$ is increased towards 90°.

102. Pipe $A$ (which can only support odd harmonics – see Eq. 17-41) has length $L_A$. Pipe $B$ (which supports both odd and even harmonics [any value of $n$] – see Eq. 17-39) has length $L_B = 4L_A$ . Taking ratios of these equations leads to the condition:

$$\left(\frac{n}{2}\right)_B = (n_{\text{odd}})_A \quad .$$

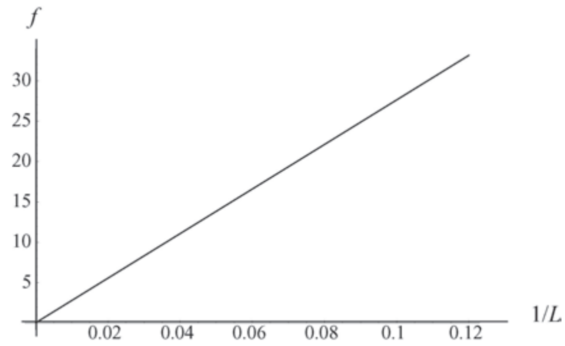
Solving for $n_B$ we have $n_B = 2n_{\text{odd}}$.

(a) Thus, the smallest value of $n_B$ at which a harmonic frequency of $B$ matches that of $A$ is $n_B = 2(1)=2$.

(b) The second smallest value of $n_B$ at which a harmonic frequency of $B$ matches that of $A$ is $n_B = 2(3)=6$.

(c) The third smallest value of $n_B$ at which a harmonic frequency of $B$ matches that of $A$ is $n_B = 2(5)=10$.

103. The points and the least-squares fit is shown in the graph that follows.




The graph has frequency in Hertz along the vertical axis and 1/*L* in inverse meters along the horizontal axis. The function found by the least squares fit procedure is $f = 276(1/L) + 0.037$. We shall assume that this fits either the model of an open organ pipe (mathematically similar to a string fixed at both ends) or that of a pipe closed at one end.

(a) In a tube with two open ends, $f = v/2L$. If the least-squares slope of 276 fits the first model, then a value of

$$v = 2(276 \text{ m/s}) = 553 \text{ m/s} \approx 5.5\times10^2 \text{ m/s}$$

is implied.

(b) In a tube with only one open end, $f = v/4L$, and we find $v = 4(276 \text{ m/s}) = 1106$ m/s $\approx 1.1\times10^3$ m/s which is more "in the ballpark" of the 1400 m/s value cited in the problem.

(c) This suggests that the acoustic resonance involved in this situation is more closely related to the $n = 1$ case of Figure 17-15(b) than to Figure 17-14.

104. (a) Since the source is moving toward the wall, the frequency of the sound as received at the wall is

$$f' = f\left(\frac{v}{v - v_S}\right) = (440\,\text{Hz})\left(\frac{343\,\text{m/s}}{343\,\text{m/s} - 20.0\,\text{m/s}}\right) = 467\,\text{Hz}.$$

(b) Since the person is moving with a speed *u* toward the reflected sound with frequency $f'$, the frequency registered at the source is

$$f_r = f'\left(\frac{v + u}{v}\right) = (467\,\text{Hz})\left(\frac{343\,\text{m/s} + 20.0\,\text{m/s}}{343\,\text{m/s}}\right) = 494\,\text{Hz}.$$

105. Using Eq. 17-47 with great care (regarding its ± sign conventions), we have

$$f' = (440 \text{ Hz})\left(\frac{340 \text{ m/s} - 80.0 \text{ m/s}}{340 \text{ m/s} - 54.0 \text{ m/s}}\right) = 400 \text{ Hz}.$$

106. (a) Let $P$ be the power output of the source. This is the rate at which energy crosses the surface of any sphere centered at the source and is therefore equal to the product of the intensity $I$ at the sphere surface and the area of the sphere. For a sphere of radius $r$, $P = 4\pi r^2 I$ and $I = P/4\pi r^2$. The intensity is proportional to the square of the displacement amplitude $s_m$. If we write $I = Cs_m^2$, where $C$ is a constant of proportionality, then $Cs_m^2 = P/4\pi r^2$. Thus,

$$s_m = \sqrt{P/4\pi r^2 C} = \left(\sqrt{P/4\pi C}\right)(1/r).$$

The displacement amplitude is proportional to the reciprocal of the distance from the source. We take the wave to be sinusoidal. It travels radially outward from the source, with points on a sphere of radius $r$ in phase. If $\omega$ is the angular frequency and $k$ is the angular wave number then the time dependence is $\sin(kr - \omega t)$. Letting $b = \sqrt{P/4\pi C}$, the displacement wave is then given by

$$s(r,t) = \sqrt{\frac{P}{4\pi C}}\frac{1}{r}\sin(kr - \omega t) = \frac{b}{r}\sin(kr - \omega t).$$

(b) Since $s$ and $r$ both have dimensions of length and the trigonometric function is dimensionless, the dimensions of $b$ must be length squared.

107. (a) The problem is asking at how many angles will there be “loud” resultant waves, and at how many will there be “quiet” ones? We consider the resultant wave (at large distance from the origin) along the $+x$ axis; we note that the path-length difference (for the waves traveling from their respective sources) divided by wavelength gives the (dimensionless) value $n = 3.2$, implying a sort of intermediate condition between constructive interference (which would follow if, say, $n = 3$) and destructive interference (such as the $n = 3.5$ situation found in the solution to the previous problem) between the waves. To distinguish this resultant along the $+x$ axis from the similar one along the $-x$ axis, we label one with $n = +3.2$ and the other $n = -3.2$. This labeling facilitates the complete enumeration of the loud directions in the upper-half plane: $n = -3, -2, -1, 0, +1, +2, +3$. Counting also the “other” $-3, -2, -1, 0, +1, +2, +3$ values for the *lower*-half plane, then we conclude there are a total of $7 + 7 = 14$ “loud” directions.

(b) The labeling also helps us enumerate the quiet directions. In the upper-half plane we find: $n = -2.5, -1.5, -0.5, +0.5, +1.5, +2.5$. This is duplicated in the lower half plane, so the total number of quiet directions is $6 + 6 = 12$.

108. The source being isotropic means $A_{\text{sphere}} = 4\pi r^2$ is used in the intensity definition $I = P/A$. Since intensity is proportional to the square of the amplitude (see Eq. 17–27), this further implies

$$\frac{I_2}{I_1} = \left(\frac{s_{m2}}{s_{m1}}\right)^2 = \frac{P/4\pi r_2^2}{P/4\pi r_1^2} = \left(\frac{r_1}{r_2}\right)^2$$

or $s_{m2}/s_{m1} = r_1/r_2$.

(a) $I = P/4\pi r^2 = (10\text{ W})/4\pi(3.0\text{ m})^2 = 0.088\text{ W/m}^2$.

(b) Using the notation $A$ instead of $s_m$ for the amplitude, we find

$$\frac{A_4}{A_3} = \frac{3.0\,\text{m}}{4.0\,\text{m}} = 0.75\,.$$

109. (a) In regions where the speed is constant, it is equal to distance divided by time. Thus, we conclude that the time difference is

$$\Delta t = \left( \frac{L-d}{V} + \frac{d}{V-\Delta V} \right) - \frac{L}{V}$$

where the first term is the travel time through bone and rock and the last term is the expected travel time purely through rock. Solving for *d* and simplifying, we obtain

$$d = \Delta t\, \frac{V(V-\Delta V)}{\Delta V} \approx \Delta t \frac{V^2}{\Delta V}.$$

(b) If we estimate $d \approx 10$ cm (as the lower limit of a range that goes up to a diameter of 20 cm), then the above expression (with the numerical values given in the problem) leads to $\Delta t = 0.8\ \mu$s (as the lower limit of a range that goes up to a time difference of 1.6 $\mu$s).

110. (a) We expect the center of the star to be a displacement node. The star has spherical symmetry and the waves are spherical. If matter at the center moved it would move equally in all directions and this is not possible.

(b) We assume the oscillation is at the lowest resonance frequency. Then, exactly one-fourth of a wavelength fits the star radius. If $\lambda$ is the wavelength and $R$ is the star radius then $\lambda = 4R$. The frequency is $f = v/\lambda = v/4R$, where $v$ is the speed of sound in the star. The period is $T = 1/f = 4R/v$.

(c) The speed of sound is $v = \sqrt{B/\rho}$, where $B$ is the bulk modulus and $\rho$ is the density of stellar material. The radius is $R = 9.0 \times 10^{-3} R_s$, where $R_s$ is the radius of the Sun ($6.96 \times 10^8$ m). Thus

$$T = 4R\sqrt{\frac{\rho}{B}} = 4(9.0\times10^{-3})(6.96\times10^{8}\ \text{m})\sqrt{\frac{1.0\times10^{10}\ \text{kg/m}^3}{1.33\times10^{22}\ \text{Pa}}} = 22\ \text{s}.$$

111. We find the difference in the two applications of the Doppler formula:

$$f_2 - f_1 = 37\ \text{Hz} = f\left(\frac{340\ \text{m/s} + 25\ \text{m/s}}{340\ \text{m/s} - 15\ \text{m/s}} - \frac{340\ \text{m/s}}{340\ \text{m/s} - 15\ \text{m/s}}\right) = f\left(\frac{25\ \text{m/s}}{340\ \text{m/s} - 15\ \text{m/s}}\right)$$

which leads to $f = 4.8 \times 10^2\ \text{Hz}$.

112. (a) We proceed by dividing the (velocity) equation involving the new (fundamental) frequency $f'$ by the equation when the frequency $f$ is 440 Hz to obtain

$$\frac{f'\lambda}{f\lambda} = \sqrt{\frac{\tau'/\mu}{\tau/\mu}} \quad \Rightarrow \quad \frac{f'}{f} = \sqrt{\frac{\tau'}{\tau}}$$

where we are making an assumption that the mass-per-unit-length of the string does not change significantly. Thus, with $\tau' = 1.2\tau$, we have $f'/440 = \sqrt{1.2}$, which gives $f' = 482$ Hz.

(b) In this case, neither tension nor mass-per-unit-length change, so the wave speed $v$ is unchanged. Hence, using Eq. 17–38 with $n = 1$,

$$f'\lambda' = f\lambda \;\; \Rightarrow \;\; f'(2L') = f\,(2L)$$

Since $L' = \frac{2}{3} L$, we obtain $f' = \frac{3}{2}(440) = 660\,\text{Hz}$.